Geophys. J. R. astr. Soc. (1982) **70**, 109–128

Computation of wave fields in inhomogeneous media – Gaussian beam approach

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Received 1981 October 20; in original form 1981 June 3

Summary. An asymptotic procedure for the computation of wave fields in two-dimensional laterally inhomogeneous media is proposed. It is based on the simulation of the wave field by a system of Gaussian beams. Each beam is continued independently through an arbitrary inhomogeneous structure. The complete wave field at a receiver is then obtained as an integral superposition of all Gaussian beams arriving in some neighbourhood of the receiver. The corresponding integral formula is valid even in various singular regions where the ray method fails (the vicinity of caustic, critical point, etc.). Numerical examples are given.

1 Introduction

An asymptotic procedure for the computation of wave fields in complicated 2-D laterally inhomogeneous media, which overcomes certain limitations of the standard ray method is suggested. Like the standard ray method, the procedure is a high frequency asymptotics. It gives, however, a good description of the wave field even in singular regions such as caustic regions, critical regions, transitions from illuminated to shadow regions, etc. The procedure also automatically removes the problem of a two-point ray tracing, which makes the standard ray method computations so cumbersome.

The procedure is based on the simulation of the wave field by a system of Gaussian beams. The wave field generated by a line source in a 2-D laterally inhomogeneous medium is decomposed into contributions, corresponding to individual rays. These contributions are evaluated along the rays by the parabolic wave equation method. The parabolic wave equation method gives solutions of the wave equation concentrated close to rays. From the physical point of view these solutions correspond to the Gaussian beams. The wave field at any point of the medium is then determined as a superposition of individual Gaussian beams passing through a vicinity of the point.

The parabolic wave equation method is closely connected with the names of Fock and Leontovich. They used the method to solve certain problems in the propagation of radio waves (Leontovich & Fock 1946; Fock 1965). The parabolic wave equation method has been applied since to many wave propagation problems, in which the waves propagating in certain preferred directions were studied. It was used to investigate mainly the radio, optical and acoustic waves, see the review paper by Tappert (1978), and also the seismic waves in seismic prospecting (Claerbout 1976). It was first applied to the investigation of the solutions of the wave equation concentrated close to rays by Babich (1968), for more details see Babich & Buldyrev (1972) and Babich & Kirpichnikova (1974). The same method was applied to elastodynamic equations by Kirpichnikova (1971).

Babich & Pankratova (1973) originally suggested to describe the wave field in the high-frequency approximation by means of an integral over all rays of the solutions concentrated close to the rays and used this integral for mathematical investigations. Basing on the paper by Babich & Pankratova (1973), Popov (1981a) suggested a new method of computation of the wave field making use of the solutions concentrated close to rays.

Results of the first attempts of using this method for computation of seismic wave fields in 2-D inhomogeneous media were presented by Popov, Pšenčík & Červený (1980), see also Červený, Popov & Pšenčík (1982). Katchalov & Popov (1981) applied this method to a wave guide problem with complicated structure of ray field. Recently Červený (1982a) obtained an expansion of a plane wave into Gaussian beams.

In this paper, we restrict ourselves to the case of a wave field generated by a line source in a 2-D laterally inhomogeneous medium. The paper is to some extent self-contained, it starts from the 2-D wave equation and gives a full derivation of the 2-D parabolic wave equation and of its solutions.

To illustrate the accuracy and efficiency of the procedure, two numerical examples are presented. In the first example, the procedure is applied to the problem of a line source in a homogeneous medium, in the other, the procedure is tested in an inhomogeneous medium in the caustic region.

It must be pointed out that in the suggested procedure, we are not interested in the Gaussian beams as in a physical reality. This subject was extensively dealt with in literature, see, e.g. Kogelnik (1965) and Felsen (1976), where other references can be also found. Here, the Gaussian beams are used only as a tool to calculate the wave fields generated by line sources in 2-D laterally inhomogeneous media.

2 Parabolic equation

We shall seek the solutions of the 2-D wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

which are concentrated close to rays. Here $V(x, z)$ is the propagation velocity, t denotes the time, x and z are the Cartesian coordinates. The quantity $u(x, z, t)$ may represent various physical quantities in different wave propagation problems.

We select an arbitrary ray Ω and introduce an orthogonal coordinate system (s, n) connected with this ray (see Fig. 1). The coordinate s measures the arclength along the ray from an arbitrary reference point, n represents a length coordinate in the direction perpendicular to Ω at s. The basis of the new coordinate system is formed by two unit vectors $\mathbf{t}$ and $\mathbf{n}$, where $\mathbf{t}$ is the unit tangent and $\mathbf{n}$ is a unit vector perpendicular to the ray Ω. The vector $\mathbf{n}$ always points to the same side of the ray. For the infinitesimal length element dr in the new

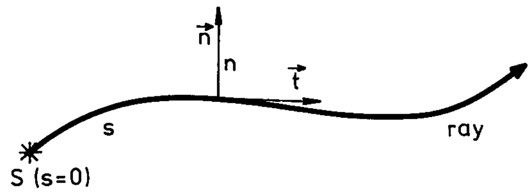

Figure 1. Ray-centred coordinate system (s, n); s is the arclength along the ray, n is the length along the line perpendicular to the ray.

coordinate frame we obtain,

$$dr^2 = h^2 ds^2 + dn^2, \tag{2}$$

where

$$h = 1 + v^{-1} v_{,n}\, n. \tag{3}$$

Equation (2) shows that the system is orthogonal, with the scale factors h, 1. In (3), we used the following notation:

$$v = V(s, 0), \qquad v_{,n} = \left[\frac{\partial V(s, n)}{\partial n}\right]_{n=0}.$$

Thus, v and $v_{,n}$ are functions of s only, not of n. Using the symbol $v(s)$ for the velocity measured directly at the ray Ω instead of $V(s, 0)$ we try to simplify the following formulae. (It would, of course, be possible to write $V(s, 0)$ instead of $v(s)$ everywhere.)

Note that in the case of the curved ray Ω the system of normals constructed to Ω will intersect at certain distances from the ray. In other words, the ray-centred coordinate system (s, n) is not regular at large distances from Ω. In the following, we shall consider only a region along Ω at which the ray-centred coordinate system (s, n) is regular and call it 'the regularity region'.

In the coordinate system (s, n) the wave equation can be rewritten as follows

$$\frac{1}{h} u_{,ss} + h u_{,nn} - \frac{h}{V^2} u_{,tt} + u_{,s}\left(\frac{1}{h}\right)_{,s} + u_{,n} h_{,n} = 0. \tag{4}$$

Here we have used the commonly accepted notation for the derivatives, $u_{,s} = \partial u/\partial s$, $u_{,ss} = \partial^2 u/\partial s^2$, etc. Similar notation will be used throughout this paper.

It is well known that the high-frequency wave field propagates mostly along rays. To find the solution of the wave equation in the case where the wave propagates mostly in some preferred direction, it is useful to use the parabolic wave equation method. We shall use the method in order to find the solutions of the wave equation concentrated close to Ω.

In the following, we shall consider only time-harmonic solutions, and denote the circular frequency by ω.

The basic step in the parabolic wave equation method is the following substitution

$$u(s, n, t) = \exp\left\{-i\omega\left[t - \int_{s_0}^{s} \frac{ds}{v(s)}\right]\right\} U(s, n, \omega). \tag{5}$$

The integral in (5) is taken along the ray Ω so that $v(s) = V(s, 0)$ is considered as a function of one coordinate s only. Inserting (5) into (4), we obtain a new differential equation for U,

$$\frac{1}{h}\left\{\left[-\frac{\omega^2}{v^2}+i\omega\left(\frac{1}{v}\right)_{,s}\right]U+\frac{2i\omega}{v}U_{,s}+U_{,ss}\right\}+hU_{,nn}+\frac{h}{V^2}\omega^2 U$$

$$+\left(\frac{i\omega}{v}U+U_{,s}\right)\left(\frac{1}{h}\right)_{,s}+U_{,n}h_{,n}=0. \qquad (6)$$

In deriving the parabolic equation, we shall assume that $n = 0\ (\omega^{-\beta})$, with $\beta = 1/2$. This assumption expresses the fact that our investigation can be restricted for large ω to a thin 'boundary layer' along Ω. The concrete value of $\beta(\beta = 1/2)$ follows from the investigation of some sample problems, see Babich & Buldyrev (1972). It is then suitable to introduce a new coordinate ν instead of $n\,[\nu = 0(1)]$ by the following relation

$$\nu = \omega^{1/2}n. \qquad (7)$$

Introducing ν into (6) we can write

$$\omega^2 h\left(\frac{1}{V^2}-\frac{1}{h^2v^2}\right)U+\omega\left[-\frac{i}{hv^2}v_{,s}U+\frac{i}{v}\left(\frac{1}{h}\right)_{,s}U+\frac{2i}{hv}U_{,s}+hU_{,\nu\nu}\right]+\omega^{1/2}U_{,\nu}h_{,n}$$

$$+\frac{1}{h}U_{,ss}+U_{,s}\left(\frac{1}{h}\right)_{,s}=0, \qquad (8)$$

where $U = U(s, \nu, \omega)$.

Equation (8) is still fully equivalent to the wave equation (1). Only now shall we start solving it asymptotically, for $\omega\to\infty$. In equation (8), we shall keep only the terms of the order of ω^γ with $\gamma \geqslant 1$, and neglect all the terms of the order ω^γ with $\gamma < 1$. Doing that, we must bear in mind, however, that several coefficients in (8) are also functions of ω. We find the expansion of these coefficients, again neglecting the terms of lower order in ω. In such a way we obtain

$$h\omega^2\left(\frac{1}{V^2}-\frac{1}{h^2v^2}\right)\sim -\frac{\omega}{v^3}\nu^2 v_{,nn},$$

where we have used the notation $v_{,nn} = [\partial^2 V(s, n)/\partial n^2]_{n=0}$. In the second term in (8), it is sufficient to consider the following approximations: $h \sim 1$, $(1/h)_{,s} \sim 0$. Then inserting the above approximations into (8) and neglecting the terms of lower order in ω, we obtain

$$\frac{2i}{v}U_{,s}+U_{,\nu\nu}-\left(\frac{1}{v^3}\nu^2 v_{,nn}+\frac{i}{v^2}v_{,s}\right)U=0, \qquad (9)$$

where $U = U(s, \nu)$ denotes in this case the leading term of the corresponding asymptotic series for $U(s, \nu, \omega)$. Note that the solution $U(s, \nu)$ depends still on the frequency ω, as $\nu = \sqrt{\omega}n$. Equation (9) is the parabolic wave equation we have sought. We can simplify it even more by the substitution

$$U(s, \nu) = \sqrt{v(s)}\ W(s, \nu). \qquad (10)$$

Inserting (10) into (9), we obtain the final form of the parabolic wave equation

$$\frac{2i}{v}W_{,s}+W_{,\nu\nu}-\frac{1}{v^3}\nu^2 v_{,nn}W=0. \qquad (11)$$

We remember that $u(s, n, t)$ is expressed in terms of W as follows:

$$u(s, n, t) = \sqrt{v(s)}\ \exp\left[-i\omega\left(t - \int_{s_0}^{s} \frac{ds}{v(s)}\right)\right] W(s, \nu), \tag{12}$$

where $W(s, \nu)$ is a solution of (11). Note again that $W(s, \nu)$ is a function of frequency ω, as $\nu = \sqrt{\omega}n$.

3 The solution of the parabolic wave equation. Gaussian beams

We shall follow Babich & Kirpichnikova (1974), and seek the solution of (11) in the following form

$$W(s, \nu) = A(s) \exp\left(\frac{i}{2}\nu^2\Gamma\right), \tag{13}$$

where $\Gamma = \Gamma(s)$ is an unknown complex-valued function. The insertion of (13) into (11) yields

$$i\left(\frac{2}{v}A_{,s} + A\Gamma\right) - A\nu^2\left(\frac{1}{v}\Gamma_{,s} + \Gamma^2 + \frac{1}{v^3}v_{,nn}\right) = 0. \tag{14}$$

We put

$$\Gamma_{,s} + v\Gamma^2 + v^{-2}v_{,nn} = 0, \tag{15}$$

and

$$A_{,s} + \tfrac{1}{2}vA\Gamma = 0. \tag{16}$$

Then (14) is satisfied and the function $W(s, \nu)$ given by (13) is the solution of the parabolic wave equation (11).

Now we shall pay attention to the solution of equation (15). From the mathematical point of view, equation (15) is an ordinary non-linear differential equation of the first order of the Riccati type that generally cannot be solved by elementary analytical methods. We shall rewrite it to the linear differential equation of the second order. We introduce a new complex-valued function q by the formula

$$\Gamma(s) = \frac{1}{vq}q_{,s}, \tag{17}$$

where $q = q(s)$. Then we get from (15) an equivalent linear differential equation of the second order for q.

$$vq_{,ss} - v_{,s}q_{,s} + v_{,nn}q = 0. \tag{18}$$

This can be rewritten into a new system of two linear differential equations of the first order putting $q_{,s} = vp$. Then we get the system

$$q_{,s} = vp, \qquad p_{,s} = -v^{-2}v_{,nn}q. \tag{19}$$

Equation (17) for Γ can be then rewritten in the following form

$$\Gamma = pq^{-1}. \tag{20}$$

To find the solutions of equation (16), it is useful to use the obvious relation $\Gamma = v^{-1} d(\ln q)/ds$, which follows from (17). Then we obtain the solution of (16) in the form

$$A(s) = \Psi q^{-1/2}(s), \tag{21}$$

where Ψ is a complex constant. The constant Ψ remains the same along the whole ray, but may be different for different rays when we denote the ray parameter by ϕ, we have $\Psi = \Psi(\phi)$.

Under $q^{1/2}(s)$ we understand here and in what follows the following function. At the initial point $s = s_0$ we choose the principal branch of the square root, i.e. $q^{1/2}(s_0) = |q(s_0)|^{1/2} \cdot \exp[(i/2) \arg q(s_0)]$, $-\pi < \arg q(s_0) \leqslant \pi$. Due to the fact that $q(s) \neq 0$ for $s \geqslant s_0$, the function $\arg q(s)$ is continuous and therefore for arbitrary $s \geqslant s_0$ we have $q^{1/2}(s) = |q(s)|^{1/2} \exp[(i/2) \arg q(s)]$.

Inserting (13), (20) and (21) into (12), and returning to the original coordinate n, we get

$$u(s, n, t) = \Psi \left[\frac{v(s)}{q(s)}\right]^{1/2} \exp\left[-i\omega\left(t - \int_{s_0}^{s} \frac{ds}{v(s)}\right) + \frac{i\omega}{2} \frac{p}{q} n^2\right], \tag{22}$$

where $p = p(s)$ and $q = q(s)$ are the solutions of equations (19).

Note that the solutions (22) are also known in the ray theory of 2-D media. The differential equations (15), (18) or (19) serve for the computations of geometrical spreading. The systems are known there as the 'dynamic ray tracing systems' or 'additional systems', i.e. additional to the standard ray tracing system, see Popov & Pšenčík (1978). In the ray methods, the functions $\Gamma(s)$, $p(s)$ and $q(s)$ are real, and the function $q(s)$ may vanish (at caustic). In the case of solutions concentrated close to rays, however, the functions $\Gamma(s)$, $p(s)$ and $q(s)$ must be complex-valued, with $\mathrm{Im}(p/q) > 0$, to guarantee the concentration of the solutions close to rays (see details later). In other words, the initial conditions for the system of equations (19) must be also complex-valued.

Let us note that solution (13) is not the only solution of the parabolic wave equation. We denote

$$W^0(s, \nu) = \frac{1}{\sqrt{q(s)}} \exp\left(\frac{i}{2} \nu^2 \frac{p}{q}\right). \tag{23}$$

It would be possible to construct from (23) an infinite number of other solutions $W^k(s, \nu)$, $k = 1, 2, \ldots$, concentrated close to rays. The solutions contain Hermite polynomials. For completeness, we present here an infinite system of linearly independent solutions of (11). It reads

$$W^k(s, \nu) = \frac{1}{\sqrt{q(s)}} \left(i \sqrt{\frac{q^*}{2q}}\right)^k H_k\left(\nu \sqrt{\mathrm{Im}\left(\frac{p}{q}\right)}\right) \exp\left(\frac{i}{2} \nu^2 \frac{p}{q}\right). \tag{24}$$

Here q^* denotes the complex conjugate of q, H_k the Hermite polynomial of the kth order, see Babich & Kirpichnikova (1974). Equation (24) can be proved by direct insertion into (11). In this paper, however, we shall not need the general equation (24), we shall use only the 'basic mode' (23).

4 Properties of Gaussian beams

In this section, we shall briefly describe some properties of Gaussian beams.

First, let us notice that the amplitude of each beam decreases exponentially with increasing distance from the ray, see (22). The exponential decrease is Gaussian, the amplitude

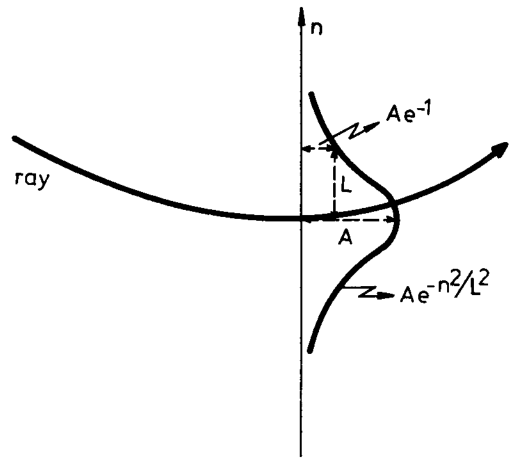

Figure 2. Amplitude profile along the cross-section perpendicular to the axis of a Gaussian beam, schematically. A, maximum amplitude; L, effective half-width of the beam.

profile along each cross-section of the beam being bell-shaped (see Fig. 2). Due to this property the solutions of the parabolic wave equation may be called Gaussian beams.

Let us now investigate more thoroughly the quantities $q(s)$ and $p(s)$. Let us denote by $\pi(s)$ the fundamental matrix of linearly independent real solutions of equations (19),

$$\pi(s) = \begin{pmatrix} q_1(s) & q_2(s) \\ p_1(s) & p_2(s) \end{pmatrix}. \tag{25}$$

In (25), each column corresponds to one solution. We shall specify the two solutions q_1, p_1 and q_2, p_2 by the initial conditions

$$\pi(s_0) = \begin{pmatrix} 1 & 0 \\ 0 & v^{-1}(s_0) \end{pmatrix}. \tag{26}$$

It is easy to show that det $\pi(s)$ does not change along the ray, so that for any s we have

$$\det \pi(s) = q_1(s)\,p_2(s) - p_1(s)\,q_2(s) = v^{-1}(s_0). \tag{27}$$

Any complex solution of (19) can be expressed in terms of two linearly independent real solutions (25) as follows

$$q(s) = z_1 q_1(s) + z_2 q_2(s), \qquad p(s) = z_1 p_1(s) + z_2 p_2(s), \tag{28}$$

where z_1 and z_2 are certain complex-valued constants, not yet specified. The constants can be selected in such a way to guarantee some important properties of Gaussian beams, see later.

Taking into account (28), we can write the expression p/q in (22) as $p/q = (z_1 p_1 + z_2 p_2)/(z_1 q_1 + z_2 q_2)$. We can separate the real and the imaginary parts of p/q and obtain a useful expression for the Gaussian beam

$$u(s, n, t) = \Psi \left[\frac{v(s)}{q(s)}\right]^{1/2} \exp\left[-i\omega[t - \tau(s)] + \frac{i\omega}{2v} K(s) n^2 - \frac{n^2}{L^2(s)}\right], \tag{29}$$

where

$$K(s) = v(s)\,\mathrm{Re}\left(\frac{p(s)}{q(s)}\right), \qquad L(s) = \left[\frac{\omega}{2}\,\mathrm{Im}\left(\frac{p(s)}{q(s)}\right)\right]^{-1/2}, \qquad \tau(s) = \int_{s_0}^{s} \frac{ds}{v(s)}. \tag{30}$$

From the physical point of view, K denotes the curvature of the phase front of the beam, $L(s)$ is some frequency-dependent effective half-width of the beam.

Let us now introduce a new complex-valued constant $\epsilon = z_1/z_2 (z_2 \neq 0)$. Then we can write (28) as follows

$$q = z_2(\epsilon q_1 + q_2), \qquad p = z_2(\epsilon p_1 + p_2).$$

For p/q we then obtain

$$\frac{p}{q} = \frac{\epsilon p_1 + p_2}{\epsilon q_1 + q_2}. \tag{31}$$

Thus, we can see that the curvature of the phase front, $K(s)$ and the effective half-width of the beam, $L(s)$ depend only on one complex-valued constant ϵ. In the case of $u(s, n, t)$, see (29), the quantity $z_2^{-1/2}$ following from the amplitude term $q^{-1/2}$ can be included into the constant Ψ, without any loss of generality. Thus, $u(s, n, t)$ depends on two complex-valued constants, ϵ and Ψ.

We can also use two real constants S_0 and L_M, both with the dimension of distance, instead of ϵ, defined in the following way,

$$\epsilon = S_0 - i\,\frac{\omega}{2v(s_0)}\,L_M^2. \tag{32}$$

Taking into account (27), we obtain

$$\mathrm{Im}\left(\frac{p}{q}\right) = \frac{\omega L_M^2}{2v^2(s_0)}\left[(S_0 q_1 + q_2)^2 + \left(\frac{\omega L_M^2 q_1}{2v(s_0)}\right)^2\right]^{-1}, \tag{33}$$

and thus for $L(s)$, see (30),

$$L(s) = \left[L_M^2 q_1^2 + \left(\frac{2v(s_0)}{\omega L_M}\right)^2 (S_0 q_1 + q_2)^2\right]^{1/2}. \tag{34}$$

Now we can specify the conditions which must be satisfied along the ray. These conditions are the following. Condition I:

$$q(s) \neq 0. \tag{35}$$

The condition guarantees the regularity of the Gaussian beam along the whole ray (with finite amplitudes at caustics). Condition II:

$$\mathrm{Im}(p/q) > 0. \tag{36}$$

The condition guarantees the concentration of the solutions close to rays.

It is easy to see that both these conditions are fulfilled for any s when

$$L_M \neq 0. \tag{37}$$

The fulfilment of condition (36) immediately follows from (33). To prove condition (35), we can replace it by an equivalent condition $q(s) \cdot q^*(s) \neq 0$, where

$$q(s) \cdot q^*(s) = \left[(S_0 q_1 + q_2)^2 + \left(\frac{\omega L_M^2 q_1}{2v(s_0)} \right)^2 \right].$$

It is possible to give a geometrical interpretation to quantities S_0 and L_M. For simplicity, we shall treat a homogeneous medium, with a constant velocity v_0. The solution of (19) with initial conditions (26) can be written as follows

$$q_1(s) = 1, \qquad q_2(s) = (s - s_0), \qquad p_1(s) = 0, \qquad p_2(s) = v_0^{-1}.$$

Formula (34) for $L(s)$ then reduces to

$$L(s) = \left[L_M^2 + \left(\frac{2v_0}{\omega L_M} \right)^2 (S_0 + s - s_0)^2 \right]^{1/2}. \tag{38}$$

This equation can be rewritten in the following form

$$\frac{L^2}{L_M^2} - \frac{4v_0^2}{\omega^2 L_M^4} (S_0 + s - s_0)^2 = 1. \tag{39}$$

We can see from (39) that function $L = L(s)$ is a hyperbola, with a minimum at $s = s_M$, where s_M follows from the relation $s_M - s_0 + S_0 = 0$. The quantity L_M represents the effective half-width of the beam at the point $s = s_M$, i.e. it is the minimum effective half-width of the beam. The quantity S_0 is the distance of the minimum of function $L = L(s)$ from the point $s = s_0$.

In the following part of this section, we shall consider beams with minimum width at $s = s_0$. In this case, we have

$$S_0 = 0, \qquad \epsilon = -i \frac{\omega}{2v(s_0)} L_M^2, \tag{40}$$

and for $L(s)$ in a homogeneous medium

$$L(s) = \left[L_M^2 + \left(\frac{2v_0}{\omega L_M} \right)^2 (s - s_0)^2 \right]^{1/2}. \tag{41}$$

In an inhomogeneous medium, we get from (34)

$$L(s) = \left[L_M^2 q_1^2 + \left(\frac{2v(s_0)}{\omega L_M} \right)^2 q_2^2 \right]^{1/2}. \tag{42}$$

We have seen that by choosing ϵ according to (32) and (37), conditions I and II are automatically satisfied. Nevertheless, we still have certain freedom in choosing the quantity L_M. We can choose it in a way which would be most suitable for numerical applications.

From the computational point of view, it is convenient to work with beams which are as narrow as possible. Thus, it is convenient to choose L_M which gives at a receiver a minimum value of the quantity $L(s)$. It simply follows from (42) that the minimum of function $L(s)$ with respect to L_M corresponds to the choice

$$L_M^{\mathrm{opt}} = \left(\frac{2v(s_0)}{\omega} \right)^{1/2} \left| \frac{q_2}{q_1} \right|^{1/2}. \tag{43}$$

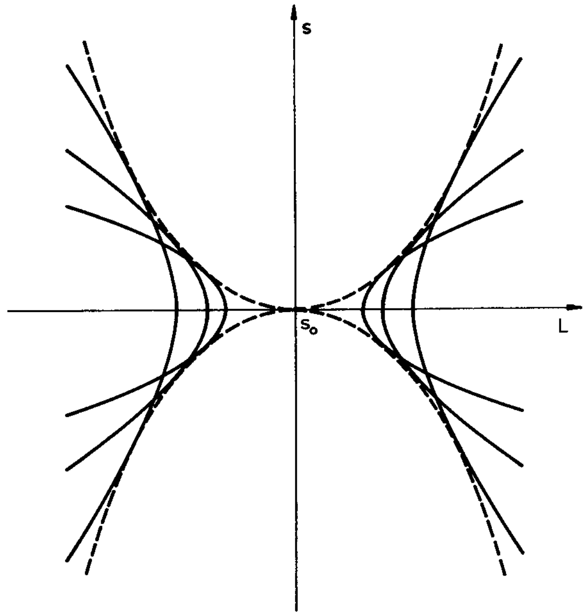

Figure 3. Three hyperbolae – Gaussian beams corresponding to various values of the effective half-width L_M of the beam at the point s_0. For small L_M, the beam is very narrow for $s = s_0$, but its width increases fast with increasing s. For large L_M, the beam is broad just from the beginning, but its width increases only slowly. The envelope of all Gaussian beams is shown by dashed lines.

For a homogeneous medium, we have

$$L_M^{opt} = \left[\frac{2v_0}{\omega}(s - s_0)\right]^{1/2}. \tag{44}$$

The meaning of the above discussed choice of L_M can be illustrated on a simple example. Let us consider a homogeneous medium, with velocity v_0. In Fig. 3, three hyperbolae (39) (Gaussian beams) corresponding to various values of L_M are shown. It is assumed that $S_0 = 0$. In this case, the narrowest parts of all the hyperbolae correspond to $s = s_0$. The width of the hyperbolae for $s \neq s_0$ is controlled by L_M. For small L_M, the beam is very narrow close to $s = s_0$, but its width increases very fast (large diffusion). For large L_M, the beam is broad just from the beginning, but its width increases only slowly. Thus, for any $s \neq s_0$, there is some optimum initial width L_M^{opt}, see (44), for which the width of the beam at s is less than for any other L_M. Inserting (44) into (41) we get the formula for the envelope of all Gaussian beams,

$$L^{env}(s) = 2\left[\frac{v_0(s - s_0)}{\omega}\right]^{1/2}.$$

This envelope is shown in Fig. 3. The narrowest hyperbola at s is that which touches the envelope.

5 Asymptotic expansion of a cylindrical wave into Gaussian beams

Let us now consider a harmonic wave field generated by a line source parallel to the y-coordinate axis. By S we denote the point at which the line source intersects the plane (x, z), by M, the observation point. We introduce the ray coordinates ϕ, ζ in the following standard way. The coordinate ϕ is the angle which specifies the initial direction of the ray at S, the coordinate ζ determines the position of a point on the ray (the arclength along the ray). We denote by ϕ_0, ζ_0 the ray coordinates of the point M and by Ω_0 the ray passing through this point.

Now we wish to express the wave field $u(\mathrm{M})$ at the point M in terms of Gaussian beams. We shall consider the Gaussian beam connected with a ray Ω, characterized by the ray coordinate ϕ, in the following form (see 22),

$$u_\phi(s, n) = \left[\frac{v(s)}{q(s)}\right]^{1/2} \exp\left\{i\omega\left[\tau(s) + \frac{1}{2}\frac{p(s)}{q(s)} n^2\right]\right\}, \tag{45}$$

where we put $\Psi = 1$, omitted the factor $\exp(-i\omega t)$ and denoted

$$\tau(s) = \int_{s_0}^{s} v^{-1}(s)\, ds.$$

It was shown in previous sections that the Gaussian beams have no singularities, neither at caustics. They are asymptotic solutions of the wave equation (1) for $\omega \to \infty$. Since the wave equation is linear, the following expression satisfies also the wave equation approximately and is regular everywhere:

$$u(\mathrm{M}) = \int_0^{2\pi} \Phi(\phi)\, u_\phi(s, n)\, d\phi. \tag{46}$$

The quantities s and n in (46) denote the coordinates of the point M in the ray-centred coordinate system connected with the ray Ω, $s = s(\phi_0, \zeta_0, \phi)$, $n = n(\phi_0, \zeta_0, \phi)$. Let us emphasize the difference between the ray coordinates ϕ_0, ζ_0, of the point M and the ray-centred coordinates s, n (connected with the ray Ω) of the same point.

If the function $\Phi(\phi)$ in (46) were known, the expression (46) could be used to evaluate the wave field at any point of the medium. We determine the function $\Phi(\phi)$ for a homogeneous medium with constant velocity $V = v_0$ by comparing the asymptotics of the integral (46) and of the exact solution of wave equation for a line source for $\omega \to \infty$. We rewrite (46) in the following form:

$$u(\mathrm{M}) = \int_0^{2\pi} F(\phi) \exp[-\omega f(\phi)]\, d\phi, \tag{47}$$

where

$$F(\phi) = \Phi(\phi)\left[\frac{v_0}{q(s)}\right]^{1/2}, \qquad f(\phi) = -i\tau(s) - \frac{i}{2}\frac{p(s)}{q(s)} n^2. \tag{48}$$

In the homogeneous medium, the coordinate ζ_0 represents the distance of the receiver M from the source S; we denote it by r. From simple geometrical considerations we obtain, see Fig. 4,

$$\tau(s) = \tau(\overline{\mathrm{SP}}) = v_0^{-1} r \cos(\phi - \phi_0), \qquad n = r \sin(\phi - \phi_0). \tag{49}$$

Taking into account the relations (31) and (49) we can write (48) as follows

$$\left.\begin{aligned} F(\phi) &= \Phi(\phi)\, v_0^{1/2} [\epsilon + r\cos(\phi - \phi_0)]^{-1/2}, \\ f(\phi) &= -\frac{i}{v_0} r\cos(\phi - \phi_0) - \frac{i}{2v_0}\frac{r^2 \sin^2(\phi - \phi_0)}{[\epsilon + r\cos(\phi - \phi_0)]} \end{aligned}\right\}. \tag{50}$$

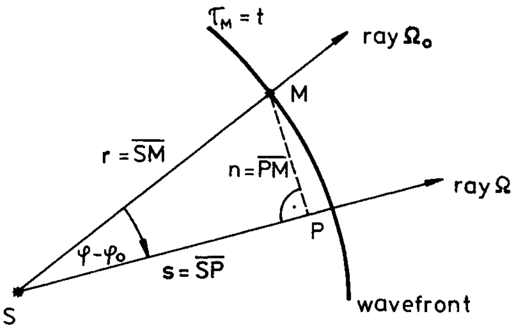

Figure 4. Definitions for the construction of the asymptotic expansion of a cylindrical wave into Gaussian beams.

We assume that ϵ does not depend on ϕ. Differentiating the function $f(\phi)$ with respect to ϕ we get

$$\frac{df(\phi)}{d\phi} = \frac{ir}{v_0}\sin(\phi-\phi_0)\left[1-\frac{r\cos(\phi-\phi_0)}{\epsilon+r\cos(\phi-\phi_0)}-\frac{1}{2}\,\frac{r^2\sin^2(\phi-\phi_0)}{v_0\,[\epsilon+r\cos(\phi-\phi_0)]^2}\right]. \tag{51}$$

It is easy to see that $df(\phi)/d\phi$ vanishes for $\phi = \phi_0$ so that $\phi = \phi_0$ is the saddle point. For the second derivative of f at $\phi = \phi_0$ we obtain

$$\left[\frac{d^2 f(\phi)}{d\phi^2}\right]_{\phi=\phi_0} = \frac{i\epsilon r}{v_0(\epsilon+r)}. \tag{51'}$$

The steepest descent approximation of the integral (46) now yields

$$u(\mathrm{M}) \sim \Phi(\phi_0)v_0\left(\frac{2\pi}{\epsilon\omega r}\right)^{1/2}\exp\left(i\frac{\omega r}{v_0}-i\frac{\pi}{4}\right). \tag{52}$$

If we compare (52) with the high-frequency asymptotics of the exact solution of the wave equation for the line source

$$u \sim -\frac{1}{4}\left(\frac{2v_0}{\pi\omega r}\right)^{1/2}\exp\left(i\frac{\omega r}{v_0}+i\frac{\pi}{4}\right), \tag{53}$$

we get for $\Phi(\phi_0)$,

$$\Phi(\phi_0) = -\frac{i}{4\pi}\left(\frac{\epsilon}{v_0}\right)^{1/2}. \tag{54}$$

Since ϵ does not depend on ϕ, we can conclude that $\Phi(\phi)$ is a complex-valued constant and equals $\Phi(\phi_0)$, given by (54). Inserting this into (46) we finally get

$$u(\mathrm{M}) \sim -\frac{i}{4\pi}\left(\frac{\epsilon}{v_0}\right)^{1/2}\int_0^{2\pi} u_\phi(s, n)\,d\phi. \tag{55}$$

Equation (55) represents an expansion of a cylindrical wave into Gaussian beams. As we can see from (55), for the evaluation of the wave field at an arbitrary point M of the medium it is necessary to determine all the Gaussian beams which give non-zero contributions to the integral (55).

As it was shown in the previous sections, the Gaussian beams can be evaluated in arbitrary inhomogeneous media. Thus, formula (55) can be used to evaluate the wave field in an arbitrary inhomogeneous medium supposing local homogeneity of the medium in a vicinity of the source. It is, however, necessary to keep in mind that the ray-centred coordinate system s, n connected with rays in inhomogeneous media is regular only in 'the regularity region' along the considered ray. Therefore, the function $u_\phi(s, n)$ in (55) should be multiplied by some windowing function which vanishes outside the regularity region. Since we wish to work with narrow beams, the situation, in which the function $u_\phi(s, n)$ will not be negligible outside the regularity region, will occur only exceptionally. In such a case, the windowing function will affect the solution (55).

Integrals of the type (55) containing the above described windowing function were first introduced and investigated by Babich & Pankratova (1973). They showed that such integrals represent a high-frequency uniform asymptotics of the investigated wave field in an inhomogeneous medium.

For more details on the expansion of a wave field generated by a line source and by a point source into Gaussian beams see Popov (1981a, b).

6 Numerical procedure

The numerical procedure for the computation of wave fields in laterally inhomogeneous media using Gaussian beams can be divided into three steps which are shortly described in the following.

In the first step, for a given model, a sufficiently dense system of rays and the quantities q_i, $p_i (i = 1, 2$, see 19) along these rays are computed. Any available standard ray tracing procedure can be used for the ray tracing. To determine the quantities q_i and p_i, a system of two linear ordinary differential equations of the first order (19) is solved for two sets of initial conditions (see 26). Routines used in the standard ray method computations for the determination of the geometrical spreading (those in which the geometrical spreading is determined by solving (19) along a ray – see, e.g. Pšenčík (1982) can be used for this purpose, after slight modifications. Some of the computed quantities are stored for the computation in the next step. The quantities to be stored are the coordinates of the points of individual rays, corresponding travel times and quantities q_i and $p_i (i = 1, 2)$.

In the second step, the coordinates s and n (see Section 2) of a receiver M (see Fig. 5) are found for each ray. These coordinates can be determined in various ways. One of them is shortly described here.

We construct a straight line through each pair of adjoining points of the ray and look for the point of intersection of this line with the perpendicular line through the receiver M.

It follows from simple geometrical considerations in Fig. 5 that the coordinate x^* of the point of intersection of the two mentioned straight lines is given by

$$x^* = [X^2 x_M + Z^2 x_i + XZ(z_M - z_i)]/(X^2 + Z^2), \tag{56}$$

where $X = x_{i+1} - x_i$, $Z = z_{i+1} - z_i$; x_i, z_i and x_{i+1}, z_{i+1} are the coordinates of the two adjoining points on the ray x_M, z_M are the coordinates of the receiver M. When x^* does not lie in the interval (x_i, x_{i+1}), the procedure is repeated for the next pair of adjoining points of the ray. In the opposite case, the coordinate $z = z^*$, the ray-centred coordinate s and the corresponding quantities q_i, $p_i (i = 1, 2)$ are determined for $x = x^*$ simply by linear interpolation (we suppose that the time step along the ray is of such a size that the corresponding part of the ray can be considered as a part of the straight line). Once the coordinate s is determined, the coordinate n can be found from the formula

$$n = [(x_M - x_i) Z - (z_M - z_i) X]/(X^2 + Z^2)^{1/2}, \tag{57}$$

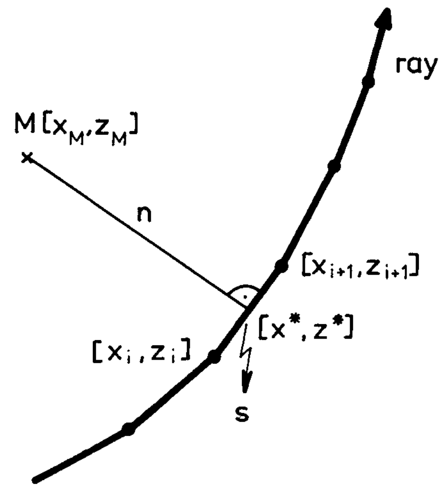

Figure 5. The determination of the coordinates s and n of the receiver M.

which again follows from simple geometrical considerations. Because now the quantities q_i and p_i are known at s, it is easy to determine the complex-valued quantities $q(s), p(s)$ at this point (see Section 4).

The final third step consists in evaluating functions $u_\phi(s, n)$ from (45) and the integral (55). The integral is replaced by a finite sum over all non-zero contributions u_ϕ. It is shown in the next section that such a replacement is quite sufficient, at least for simple models considered in Section 7.

7 Numerical examples

The method described in the previous sections was numerically tested on various models of media. These tests served two main purposes. First, to estimate the accuracy of the method by comparing its results with the results of other accurate or sufficiently accurate methods. Second, to find answers to various questions concerning the numerical realization of the method. For example, it is necessary to find how many rays should be used for evaluating the integral (55), how dense the system of rays should be, how to approximate the integral (55), etc.

The preliminary results of some of the mentioned tests are presented in the following.

7.1 REGULAR WAVE FIELD IN A HOMOGENEOUS MEDIUM

In this example we compare the results of the Gaussian beam approach (GBA) with the results of the asymptotic ray theory (ART) for the case of line source in a homogeneous medium:

$$u_{\mathrm{ART}} = C(2\pi v_0/\omega r)^{1/2} \exp(i\omega r/v_0),$$

with $C = -(4\pi)^{-1} \exp(i\pi/4)$ (see 53).

Some characteristic results are shown in Tables 1–5. In these tables the results of both the GBA and the ART are multiplied by the constant $C^{-1} = -4\pi \exp(-i\pi/4)$. The following notation is used in the tables. A_{GBA} (A_{ART}) and Φ_{GBA} (Φ_{ART}) are the amplitude and the phase of the GBA (ART) solutions. The distance between the source and receiver is

Table 1. The effect of the wavelength λ on the GBA results. Explanation: A_{GBA} (A_{ART}) and Φ_{GBA} (Φ_{ART}) are the amplitude and the phase of the GBA (ART) solutions; L_M is the effective half-width of the beam at the source, r denotes the distance between the source and receiver, $\Delta\phi$ is a step in the initial angles of the rays used for the construction of the GBA solution; the initial angles ϕ are measured in both directions from the ray connecting the source and receiver up to the value ϕ_{max} (thus the rays form a cone with the angle $2\phi_{max}$); N is a number of rays used for the construction of the GBA solution.

λ	L_M	A_{GBA}	Φ_{GBA}	A_{ART}	Φ_{ART}
0.62	4.5	0.0793	0.9742	0.0793	0.9736
6.3	14.1	0.2508	−0.5316	0.2507	−0.5310
62.8	44.7	0.8095	−2.5046	0.7927	−2.5664

$r = 100$, $\Delta\phi = 0.01$, $\phi_{max} = 0.5$, $N = 101$.

Table 2. The effect of epicentral distance r on the GBA results. For notations see Table 1.

r	L_M	A_{GBA}	Φ_{GBA}	A_{ART}	Φ_{ART}
100	14.1	0.2506	−0.5285	0.2507	−0.5310
200	20.0	0.1772	−1.0606	0.1773	−1.0619
300	24.5	0.1747	−1.5923	0.1747	−1.5929
400	28.3	0.1253	−2.1233	0.1253	−2.1239
500	31.6	0.1121	−2.6542	0.1121	−2.6548

$\lambda = 6.3$, $\Delta\phi = 0.04$, $N = 41$, $\phi_{max} = 0.8$.

Table 3. The effect of the width of the cone of rays, used for the GBA computations, on the GBA results. A_{min}/A_{max} is the ratio of the amplitude contribution on the ray ϕ_{max} to the amplitude contribution on the ray connecting the source and receiver. For notations see Table 1.

N	ϕ_{max}	A_{GBA}	Φ_{GBA}	A_{min}/A_{max}
101	0.5	0.2506	−0.5283	0.0016
81	0.4	0.2508	−0.5316	0.0169
61	0.3	0.2555	−0.5170	0.1031
41	0.2	0.2449	−0.3968	0.3673
21	0.1	0.1612	−0.2230	0.7795

$\lambda = 6.3$, $r = 100$, $\Delta\phi = 0.01$, $L_M = 14.1$, $A_{ART} = 0.2507$, $\Phi_{ART} = -0.5310$.

Table 4. The effect of the density of the rays, used for the GBA computations, on the GBA results. For notations see Table 1.

$\Delta\phi$	N	A_{GBA}	Φ_{GBA}
0.01	81	0.2508	−0.5316
0.02	41	0.2507	−0.5313
0.04	21	0.2505	−0.5306
0.06	15	0.2504	−0.5290
0.08	11	0.2504	−0.5293
0.1	9	0.2504	−0.5286
0.2	5	0.2519	−0.5441
0.4	3	0.3299	−0.1102

$\lambda = 6.3$, $r = 100$, $\phi_{max} = 0.4$, $L_M = 14.1$, $A_{ART} = 0.2507$, $\Phi_{ART} = -0.5310$.

Table 5. The effect of the effective half width of the beam at the source on the GBA results. For notations see Tables 1 and 3.

L_M	A_{GBA}	Φ_{GBA}	A_{min}/A_{max}
44.7	0.2684	−0.4747	0.4719
20.0	0.2493	−0.5251	0.0441
14.1	0.2508	−0.5316	0.0169
10.0	0.2505	−0.5104	0.0328
7.1	0.2389	−0.4770	0.1299
5.8	0.2246	−0.4629	0.2460
5.0	0.2113	−0.4568	0.3477
1.4	0.0794	−0.5576	0.9533

$\lambda = 6.3$, $\Delta\phi = 0.01$, $N = 81$, $\phi_{max} = 0.4$, $A_{ART} = 0.2507$, $\Phi_{ART} = -0.5310$.

denoted by r, the ray connecting these two points being called the central ray. The quantity $\Delta\phi$ is a step in the initial angles of the rays used for the construction of the GBA solution. The initial angle ϕ is measured in both directions from the central ray up to the value ϕ_{max}. Thus the wave field is investigated in a cone with the angle $2\phi_{max}$. The quantities ϕ, $\Delta\phi$ and ϕ_{max} are given in radians. N is a number of rays covering such a cone, $N = 2\,\phi_{max}/\Delta\phi + 1$. The quantities r, λ (wavelength) and L_M have the dimensions of length. No units are written for these quantities in the tables; all the units must be, of course, compatible.

It is evident from Tables 1 and 2 that the GBA procedure works sufficiently well even for large wavelengths (low frequencies) and large epicentral distances. It is only necessary to consider sufficiently wide cones of rays.

The effect of the width of the cone on the GBA results is shown in Table 3. In column A_{min}/A_{max}, the ratios of amplitude contribution on the ray ϕ_{max} to the amplitude contribution on the central ray are given. It follows from Table 3 that for $A_{min}/A_{max} = 0.1031$ the difference between A_{GBA} and A_{ART} is less than 2 per cent. Even for $A_{min}/A_{max} = 0.3673$ this difference is less than 3 per cent, but the difference between corresponding phases is larger.

How the density of rays inside the cone affects the GBA results is illustrated by the results shown in Table 4. It can be seen that only five beams can approximate quite well the wave field at the epicentral distance of 100 km and frequency ~1 Hz (for velocity v_0 about 6 km s^{-1}).

The comparison of the results in Table 5 obtained for different values of the quantity L_M is also interesting. The row corresponding to the optimum value of L_M giving a minimum width of the beam (see 44 and the following discussion) is underlined. When L_M differs from this optimum value, the beams become wider so that the cone of rays used in Table 5 is not sufficiently wide for them (see the values in column A_{min}/A_{max}) and the results are distorted. Thus, when we evaluate the wave field at a certain epicentral distance, it is very convenient to choose $L_M = [2\omega^{-1}v(s_0)\,|q_2 q_1^{-1}|]^{1/2}$ (see Section 4).

7.2 WAVE FIELD IN THE VICINITY OF A CAUSTIC

To test the GBA method in a singular region, the method was used to evaluate the wave field in the vicinity of a caustic. A simple model consisting of a homogeneous layer over a layer with a constant gradient used for this purpose is shown in Fig. 6. The corresponding ray diagram of the investigated refracted wave is shown in Fig. 7. In this figure the caustic which

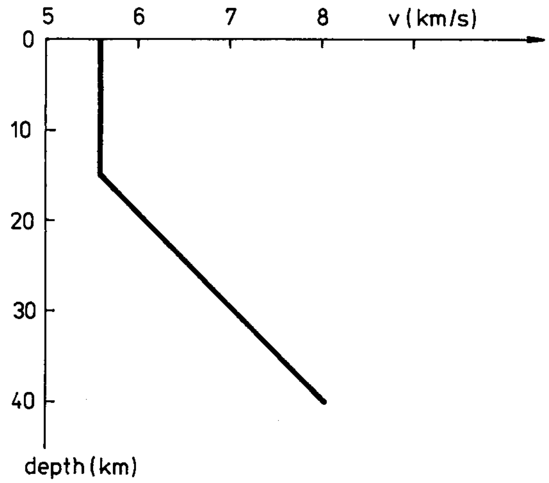

Figure 6. Velocity model used for computations.

intersects the surface at the epicentral distance of 120 km is clearly expressed. The caustic separates the illuminated zone from the shadow zone.

The mentioned model of a medium was used earlier for testing various modifications of the ART method (Červený, Molotkov & Pšenčík 1977). Here, these modifications are used for comparison with the GBA results. Slight differences should be expected between the results, for a point source was used in the ART modification while a line source is considered in the GBA method. To get the GBA results corresponding to the point source, a standard procedure from the ART method was used, the GBA results were multiplied by

$$-(8\pi\omega/v_0 r)^{1/2}\exp(-i\pi/4),$$

where r is the epicentral distance.

To make the numerical procedure more effective, the optimization of the width of Gaussian beams based on equation (43) was performed. First, the quantities q_1, q_2 specified by the initial conditions (26) were determined for the ray passing most closely to the receiver under consideration. Then, using q_1, q_2 the optimum values of L_M (see 43) and ϵ

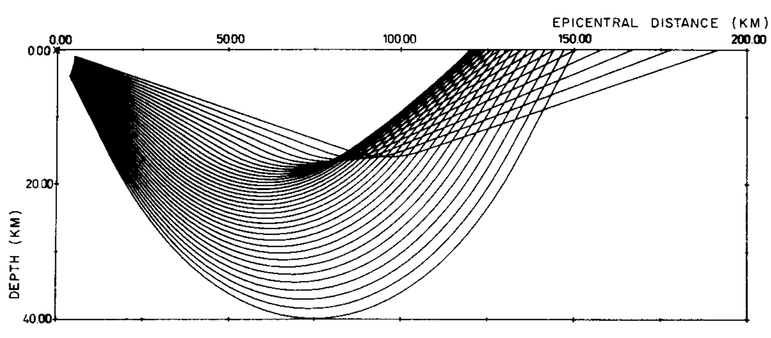

Figure 7. Ray diagram of a refracted wave in the model shown in Fig. 6.

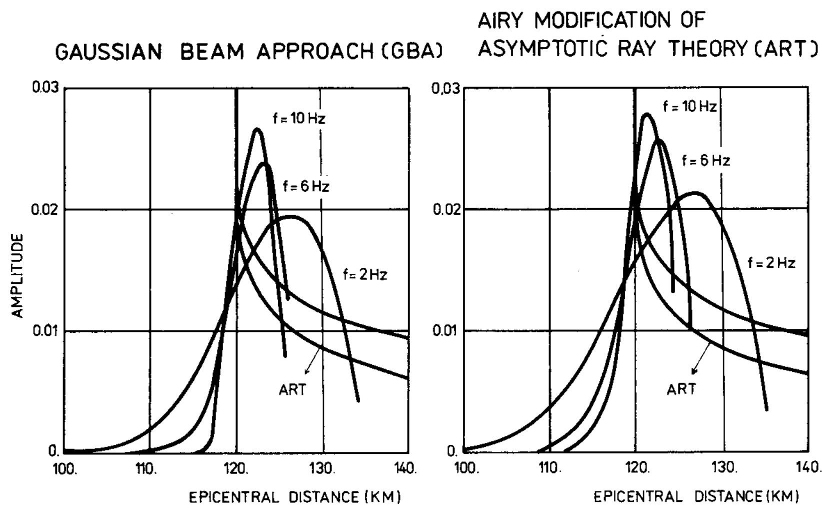

Figure 8. Amplitude–distance curves of a refracted wave in the vicinity of caustic for the model shown in Fig. 6. The curves in the left picture were computed by the GBA method, the curves in the right picture by the Airy modification of the ART method. Amplitude–distance curves computed by the standard ray method and denoted ART are shown for comparison.

(see 32) were found. The determination of the functions $u_\Phi(s, n)$ for different Φ and the evaluation of the integral (55) was then straightforward.

The results obtained by the Airy modification of the ART (right) and the GBA (left) for different frequencies f are presented in Fig. 8. Except for some slight differences discussed above, the results coincide. For comparison, the standard ray method amplitude–distance curves (denoted by the ART) are also shown. The latter can be observed only in the illuminated zone, they increase to infinity when approaching the caustic where the ART method is not applicable. On the contrary, the curves obtained by the GBA and modified ART give finite non-zero values at all points in the vicinity of the caustic including the shadow zone. Moreover, they display a frequency-dependent behaviour. In the vicinity of the caustic both methods give satisfactory results. Unlike the modification of the ART, the GBA method can be used without any change in other singular regions, no matter how complicated they are.

8 Concluding remarks

The asymptotic procedure proposed in the paper is applicable to any 2-D laterally inhomogeneous media with curved interfaces.

Compared with the standard ray method, the procedure has some important advantages. The most important of them are listed below:

(1) The procedure is applicable even in various regions, in which the standard ray method fails (e.g. in caustic regions, critical regions, transitions from illuminated to shadow zones).

(2) To determine the seismic wave field at a specified point of a medium, it is not necessary to perform two-point ray tracing, which makes standard ray computations so cumbersome (especially in 3-D models).

(3) In the standard ray method, the velocity distribution and interfaces must be sufficiently smooth, without fictitious discontinuities and oscillations, which can greatly

influence the results. It is expected that the effect of these factors will be less in the GBA method.

(4) Since the Gaussian beams are frequency dependent, it would be easy to consider also certain frequency-dependent phenomena in the wave field (e.g. absorption, frequency-dependent transmission and reflection coefficients, etc.)

In application, it is convenient to evaluate the integral (55) numerically as it was done in the presented numerical examples. The integral, however, can be also evaluated approximately by various asymptotic methods, even in singular regions. Of course, the most precise results are always obtained by a direct numerical evaluation of integral (55).

In this paper only harmonic solutions were considered. It is, however, not difficult to apply the GBA procedure also to the non-stationary wave fields and to the computation of synthetic seismograms. Three different approaches how to do it are described by Červený (1982b). These computations are expected to be even faster than the corresponding ART computations since the GBA method does not include two-point ray tracing.

Here we have concentrated on the computation of wave fields in models without interfaces. The Gaussian beam approach, however, can be generalized even for media with interfaces. For scalar wave fields described by the wave equation, the problem of reflection/transmission of Gaussian beams at curved interfaces was solved for example in the book by Babich & Buldyrev (1972). Thus, it is possible to consider also multiply reflected waves in the above described procedure (see Popov 1981b). Similar generalization for elastodynamic equation (seismic waves) will be given elsewhere.

The GBA procedure can be also applied to the 3-D wave equation and 3-D elastodynamic equation. Kirpichnikova (1971) derived corresponding formulae for Gaussian beams for the latter case. As in the 2-D case, the GBA procedure is expected to be faster than the ART, because the time-consuming 3-D two-point ray tracing will not be involved.

References

Babich, V. M., 1968. Eigenfunctions, concentrated in the vicinity of closed geodesics, in *Mathematical Problems of Theory of Propagation of Waves,* Vol. 9, pp. 15–63, Nauka, Leningrad (in Russian).

Babich, V. M. & Buldyrev, V. S., 1972. *Asymptotic Methods in Problems of Diffraction of Short Waves,* Nauka, Moscow (in Russian).

Babich, V. M. & Kirpichnikova, N. J., 1974. *Boundary Layer Method in Diffraction Problems,* Leningrad University Press (in Russian, English translation by Springer-Verlag, 1980).

Babich, V. M. & Pankratova, T. F., 1973. On discontinuities of the Green function of mixed problem for wave equation with variable coefficients, in *Problems of Mathematical Physics,* Vol. 6, pp. 9–27, Leningrad University Press (in Russian).

Červený, V., 1982a. Expansion of a plane wave into Gaussian beams, *Studia geophys. geod.,* **26,** in press.

Červený, V., 1982b. Computation of synthetic seismograms for 1D and 2D inhomogeneous media. Numerical methods in the interpretation of seismic data, *Suzdal* 1980, in press (in Russian).

Červený, V., Molotkov, I. A. & Pšenčík, I., 1977. *Ray Method in Seismology,* Karlova Univerzita, Praha.

Červený, V., Popov, M. M. & Pšenčík, I., 1982. Computation of seismic wave fields in laterally inhomogeneous crustal structures – Gaussian beams approach, *Proc. XVII General Assembly of the European Seismological Commission,* Budapest, 1980, Hungarian Geophysical Society, in press.

Claerbout, J. F., 1976. *Fundamentals of Geophysical Data Processing,* McGraw-Hill, New York.

Felsen, L. B., 1976. Complex-source-point solutions of the field equations and their relation to the propagation and scattering of Gaussian beams, *Inst. Nazionale Alta Matem., Symp. Math.,* **18,** 39–56.

Fock, V. A., 1965. *Electromagnetic Diffraction and Propagation Problems,* Pergamon Press, New York.

Katchalov, A. P. & Popov, M. M., 1981. The application of the Gaussian beams summation method to the computation of wave fields in the high-frequency approximation, *Dokl. Akad. Nauk USSR,* **258** (5), 1097–1100 (in Russian).

Kirpichnikova, N. J., 1971. Construction of solutions concentrated close to rays for the equations of elasticity theory in an inhomogeneous isotropic space, in *Mathematical Problems of Theory of Diffraction and Propagation of Waves,* Vol. 1, Nauka, Leningrad (in Russian, English translation by American Mathematical Society, 1974).

Kogelnik, H., 1965. On the propagation of Gaussian beams of light through lens-like media including those with a loss or gain variation, *Appl. Opt.,* **4,** 1562–1569.

Leontovich, M. A. & Fock, V. A., 1946. Solution of the problem of propagation of electromagnetic waves along the Earth's surface using parabolic equation method, *ZETF,* **16,** 557–573 (in Russian).

Popov, M. M., 1981a. A new method of computation of wave fields in the high-frequency approximation, *Preprint LOMI AN SSSR, E-I-81,* Leningrad.

Popov, M. M., 1981b. A new method of computation of wave fields in high-frequency approximation, in *Mathematical Problems of Theory of Propagation of Waves,* Vol. 11, pp. 195–216, Nauka, Leningrad (in Russian).

Popov, M. M. & Pšenčík, I., 1978. Computation of ray amplitudes in inhomogeneous media with curved interfaces, *Studia geophys. geod.,* **22,** 248–258.

Popov, M. M., Pšenčík, I. & Červený, V., 1980. Uniform ray asymptotics for seismic wave fields in laterally inhomogeneous media, *Prog. Abstr. XVII General Assembly of the ESC,* Budapest, 1980, p. 143. Hungarian Geophysical Society.

Pšenčík, I., 1982. Program for the computation of kinematic and dynamic parameters of seismic body waves, in 2-D laterally inhomogeneous media with curved interfaces, in *Programs for the Interpretation of Seismic Observations,* Vol. 3, Nauka, Leningrad, in press (in Russian).

Tappert, F. D., 1978. The parabolic approximation method, in *Wave Propagation and Underwater Acoustics,* Lecture Notes in Physics, **70,** 224–287, Springer-Verlag, Berlin.