P6317 Assignment II

Due, Tuesday, March 15, 2022

1) Consider sound travelling across an air-water interface. At the interface, the particle speed and the pressure must match.

   a) [5] For a given velocity (at the boundary), compare the pressure anomalies far from the interface in the air and in the water. Assume, $\rho_{air} = 1.39$ kg/m^3, $C_{air} = 335$ m/s, $\rho_{water} = 1000$ kg/m^3, and $C_{water} = 1500$ m/s.

   b) [5] Compare how a sound source designed for use in the air would perform if used underwater. And, how would a sound source designed for water perform if used in the air. Assume that the transducer drives the velocity at the transducer face at a fixed value whether in water or in air. How reasonable do you think the last assumption is (justify your answer).

2) a) [5] Twentyone sources are equally spaced over 2.0 m along the z axis. The sources have equal amplitudes (a/21), and signal frequency f = 15,000 Hz. Calculate and graph the source pattern far from the source over a range of $-.2 \leq \sin\phi \leq .2$.

   b) [5] Repeat the calculation and graph but this time assume a continuous distribution of sources of strength a/2 m^{-1}. Compare the two plots.

3) a) [8] For a typical ocean sound channel, the sound speed initially decreases with depth and then increases with depth so that the sound speed at the ocean bottom is greater than that at the surface. In this situation, the limiting rays that remain in the sound channel (without reflecting at the bottom or surface) are determined by those that are horizontal at the surface. Consider a source somewhere in the sound channel where the sound speed is $C_s = C(0) - \Delta C$ ($C(0)$ is the sound speed at the surface). For this source, demonstrate that the maximum angle that a ray can have with the horizontal and still remain trapped by the sound channel is given by,

$$\theta_{max} \simeq \sqrt{2\Delta C / C_s} \tag{1}$$

   (Hint: use snells law, assume θ_{max}, and ΔC are small).

3) a) [2] How would Equation (1) be changed if the sound speed at the bottom was less than at the surface.

4) [10] Convergence zones occur when sound rays starting out at or near the surface, refract downward, through a sound channel and then back up to the surface. Consider a simplified sound speed profile where the sound channel axis is defined as depth $z = 0$ and for which the sound speed above and below can be described by the equations $C = C_0 + g'z$ and $C = C_0 - gz$ respectively, $z = h'$ at the surface, and $z = -h$ at the bottom, $C(h') = C_1'$, $C(h) = C_1$, and $C_1 > C_1'$.

For a source located at the surface, in terms of C and g, at what distance will the horizontal ray reappear at the surface. Evaluate the result numerically if $C_0 = 1475$ m/s, $C_1' = 1505$ m/s, $C_1 = 1530$ m/s, $h' = 400$ m, and $h = 3200$ m. (Recall that ray paths describe the arc of a circle in a constant sound speed gradient ocean).

1)

a) Here we can use $p = \rho c u$

we have that u is the same in air/water

so

$$p_{air} = \rho_{air} \cdot c_{air} \cdot u$$

$$p_w = \rho_w \cdot c_w \cdot u$$

$\rho_a = 1.39 \ kg/m^3$

$\rho_w \sim 1000 \ kg/m^3$

$c_{air} = 335 \ m/s$

$c_w \sim 1500 \ m/s$

$$\frac{p_a}{\rho_a c_a} = \frac{p_w}{\rho_w c_w}$$

$$\frac{p_a}{p_w} \sim \frac{\rho_a c_a}{\rho_w c_w} = \frac{1.39 \cdot 335}{1000 \cdot 1500} = 3.1 \cdot \times 10^{-4}$$

→ Pressure anomaly in air is 4 orders of magnitude smaller than in water!

b) If the transducer could drive the velocity of the material air/water at the same speed

i) air transducer in water; if the transducer was intended to create pressure fluctuations p', in water it would create fluctuations $10^4 \cdot p'$! Much bigger!

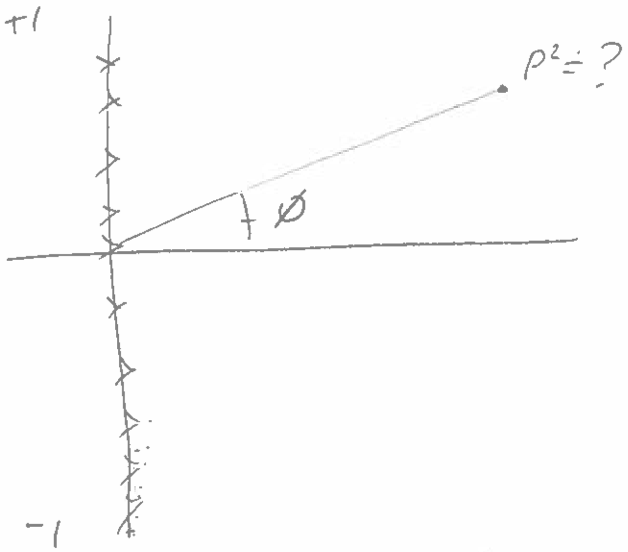
ii) water transducer in air: designed for p' would create fluctuations $10^{-4} p'$!

cont ↓

The assumption of the transducer driving the fluid with the same velocity is flawed! If you consider the energy density (intensity) which goes as $p^2/\rho c$ for a given energy in air the pressure fluctuations need be much smaller than those in water; the air transducer would not move at all in water, and the water transducer would feel no resistance in air.

3a)

21 sources over a 2m long array, each source has strength $a/21$



need to sum up terms

$$p = \frac{a}{21} \cdot \sum_{j=1}^{21} e^{i k z_j \sin\phi}$$

$$z_j = -1 + \frac{j-1}{20} \cdot 2 \Big|_{j=1-21}$$

$$e^{i k z_j \sin\phi} = \cos(k z_j \sin\phi) + i(\sin k z_j \sin\phi)$$

$$p = \frac{a}{21} \cdot \left(\Sigma \cos + i \,\Sigma \sin\right)$$

$$|p|^2 = \frac{a}{21} \cdot (\Sigma \cos)^2 + (\Sigma \sin)^2$$

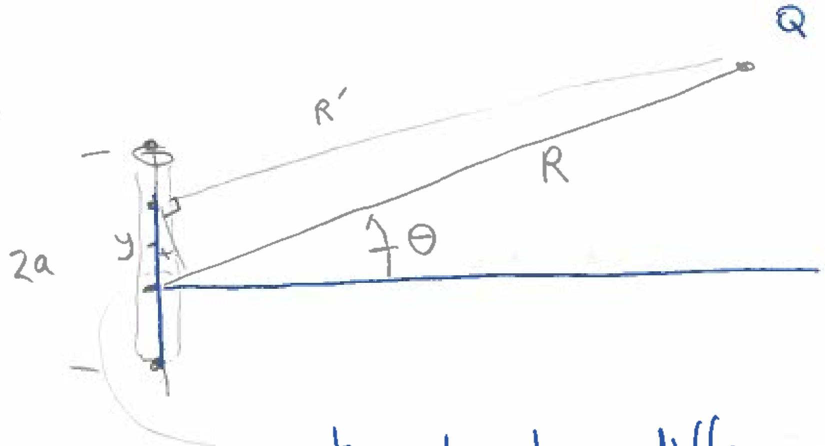
$$= \frac{S}{2a} \frac{-e^{-ika\sin\theta} + e^{ika\sin\theta}}{+ik\sin\theta}$$

$$\boxed{= \frac{S}{a} \frac{\sin(ka\sin\theta)}{k\sin\theta}}$$

Can use same approach to find source pattern for circular transducer ...

But now, the strength of each element is not simply dy, but is modulated by the changing width ....

Solution to Q 3 ass 2 (2006)



path length difference in $y \sin\theta$

Contribution at Q will be

$$\delta P = \delta S \, e^{i(kR' - \omega t)}$$

source contribution

$$R' = R - y\sin\theta$$

$$\delta P = \delta S \, e^{i\,-ky\sin\theta} \cdot e^{i(kR-\omega t)}$$

What for δs? $\rightarrow \dfrac{S\,dy}{a}$

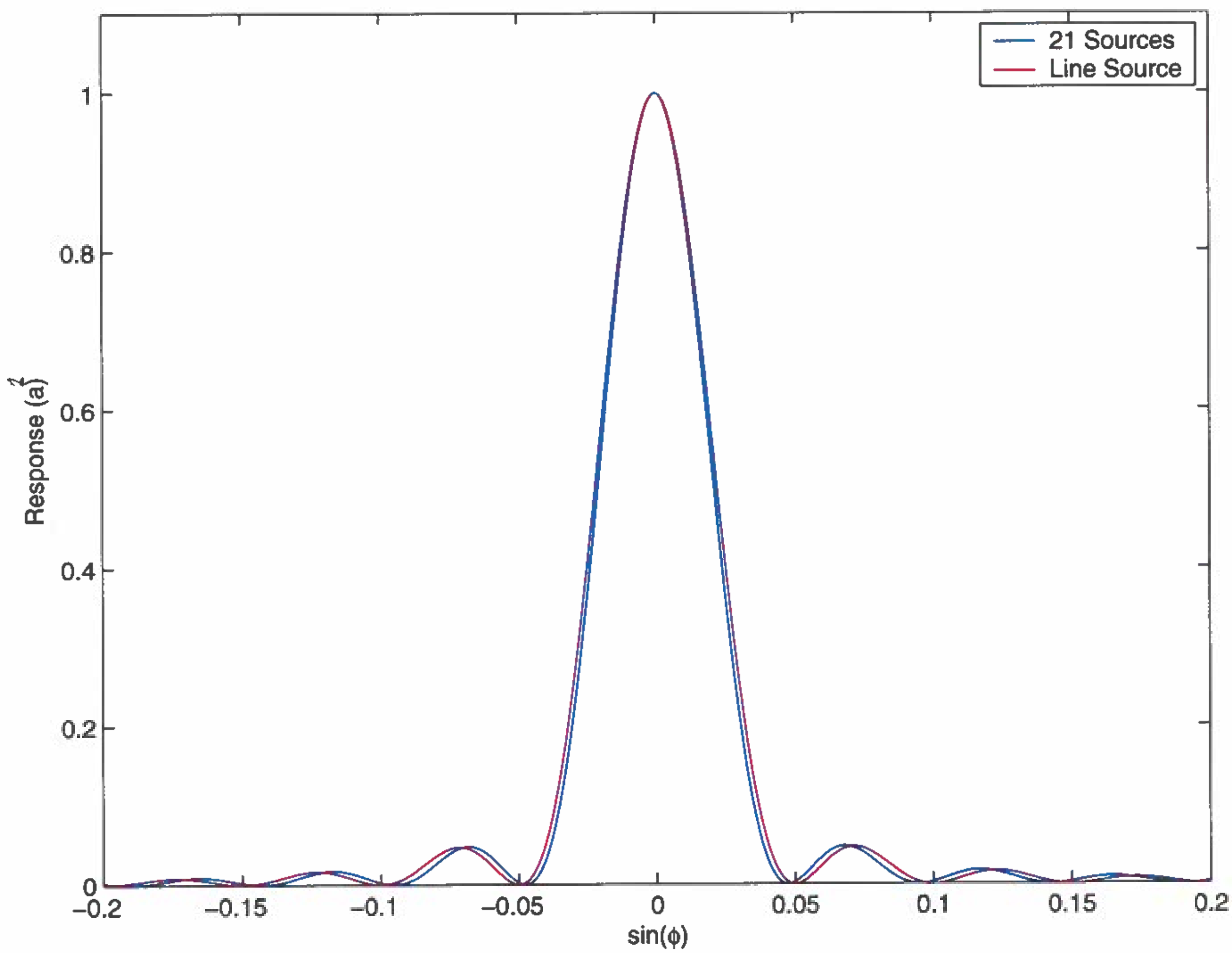
$$\delta P = \frac{1}{2a}\,dy \quad e^{-(iky\sin\theta)} \quad \boxed{e^{i(kR-\omega t)}}$$

Just a constant ignore

$$P = \frac{S}{2a}\int_{-a}^{a} e^{-(iky\sin\theta)}\,dy$$

$$= \frac{S}{2a}\left.\frac{e^{-iky\sin\theta}}{-ik\sin\theta}\right|_{-a}^{a}$$

Ass 2, Question 3



```
%  first the 21 source solution

f = 15000;
C = 1500;
lambda = C/f;
k = 2*pi/lambda;


j = 1:21;
zj = -1 + (j-1)./20.*2;


sphi = -.2:.001:.2;

P2 = (1/21)^2 * sum(cos(k*zj'*sphi)).^2 + sum(sin(k*zj'*sphi)).^2 ;
plot(sphi,P2)
hold on

w = 2;
sinc = sin(k*w*sphi/2)./(k*w*sphi/2);
sinc2 = sinc.^2;
plot(sphi,sinc2,'r')

xlabel('sin(\phi)')
ylabel('Response (a)')
title('Ass 2, Question 3')
axis([-.2 .2 0 1.1])
legend('21 Sources','Line Source')
hold off
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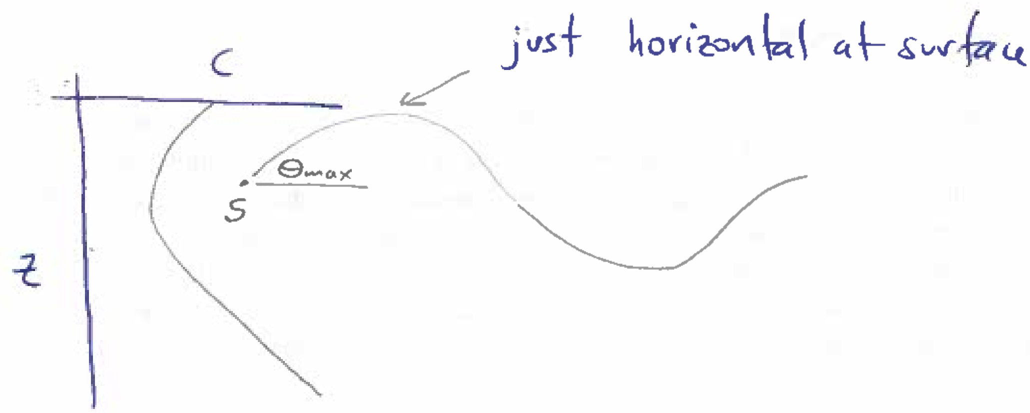
1)



Sound speed at source $\quad c_s = c(0) + \Delta c$

The limiting angle/ray is just horizontal at the surface ...

$$\frac{\cos\theta_{max}}{c_s} = \frac{\cos\theta_{surface}}{c(0)}$$

$$\theta_{surface} = 0 \longrightarrow \frac{\cos\theta_{max}}{c_s} = \frac{1}{c(0)}$$

$$\frac{c_s}{c(0)} = \cos\theta_{max} = \frac{c_s}{c_s + \Delta c}$$

if θ_{max} is small

$$1 - \frac{\theta_m^2}{2} + \text{hot} = \frac{c_s}{c_s + \Delta c} = \frac{1}{1 + \frac{\Delta c}{c_s}} \simeq 1 - \frac{\Delta c}{c_s}$$

$$\cancel{1} - \frac{\Delta c}{c_s} = \cancel{1} - \frac{\theta_{max}^2}{2}$$

$$\boxed{\sqrt{\frac{2\Delta c}{c_s}} = \theta_{max}}$$

(~~Δc must be ≥ 0~~!)

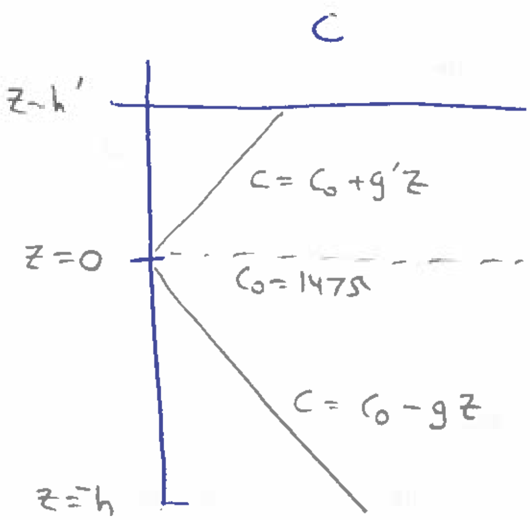
1b) If the sound speed at the bottom was less than at the surface, consider the max angle as that giving a horizontal ray at the bottom. Instead of using $c(0)$ = surface sound speed, use $c(0)$ = bottom sound speed.

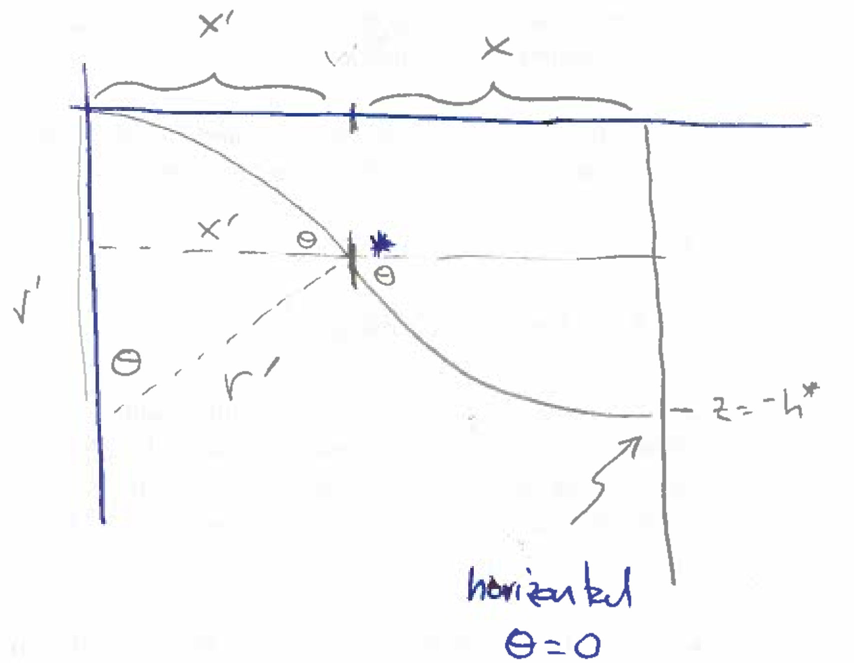
2)

Consider a ray starting out horizontal at the surface; follow this path as it goes along – recall for constant sound speed gradient the ray paths are circular arcs with radius

$$r = \frac{c_s}{g \cos\theta_s} \quad \text{for} \quad c = c_0 + gz$$

and an initial ray angle at sound speed c_s of θ_s.





Total distance to convergence zone will be $(x' + x)\,2$

$$x' = r'\sin\theta = r'\sin\left[\operatorname{acos}\frac{r'-h'}{r'}\right]$$

similarly

$$x = r\sin\theta = r\sin\left[\operatorname{acos}\frac{r-h^*}{r}\right] = r\sin\left[\operatorname{acos}\frac{r'-h'}{r'}\right]$$

because of symmetry at *

2 cont

$$r' = \frac{C_{surface}}{g'} = \frac{C_s \cdot h'}{(C_s - C_o)} = \frac{1505 \cdot 400}{1505 - 1475} = 20{,}066$$

$r' \sin\theta = x' = 3986$ km

$$r = \frac{C_o}{g \cos\theta}$$

$$\theta = a\cos\left[\frac{r' - h'}{r'}\right] = a\cos \frac{20{,}066 - 400}{20{,}066} = 11.46^\circ$$

$$r = \frac{1475}{(1530 - 1475)} \cdot \frac{3200}{\cos 11.46^\circ} = 87{,}563$$

$r \sin\theta = x = 17.37$ km

$$x_{tot} = 2(r + r')\sin 11.46^\circ = 42{,}768 \simeq \boxed{\underline{43 \text{ km}}}$$