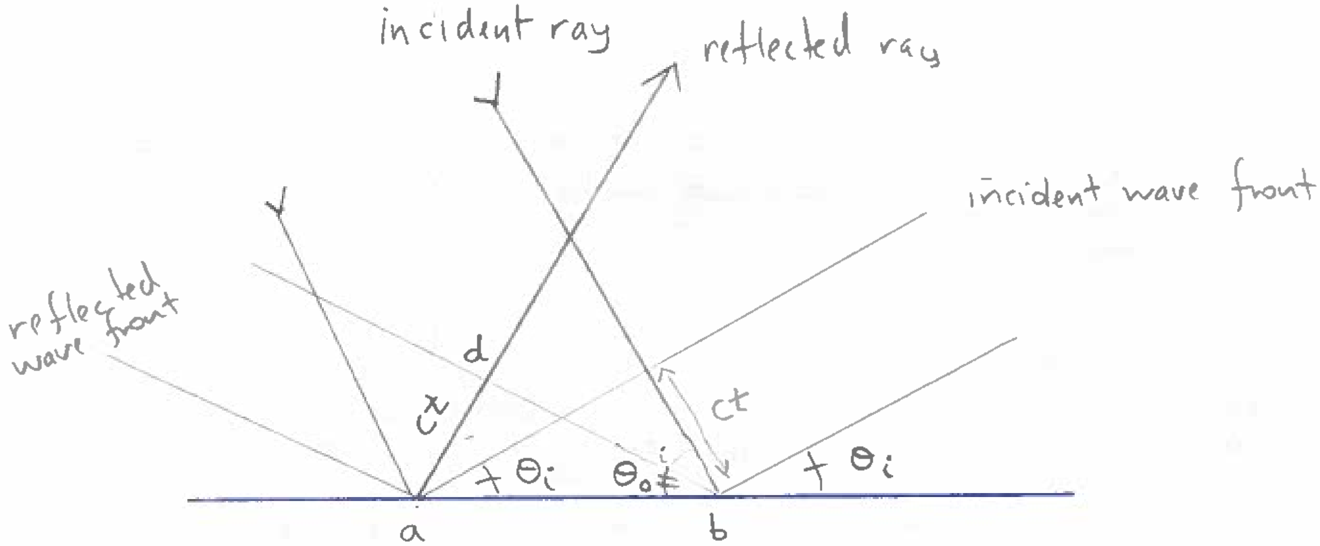
11



At some time, a phase front of the incident plane wave encounters the surface at "a" this point becomes a Huygen's source point. Wavelets radiate from "a" and at some time t later they have traveled a distance ct. Also in that time, the incident wave has travelled a distance ct so that it now encounters the interface at "b".

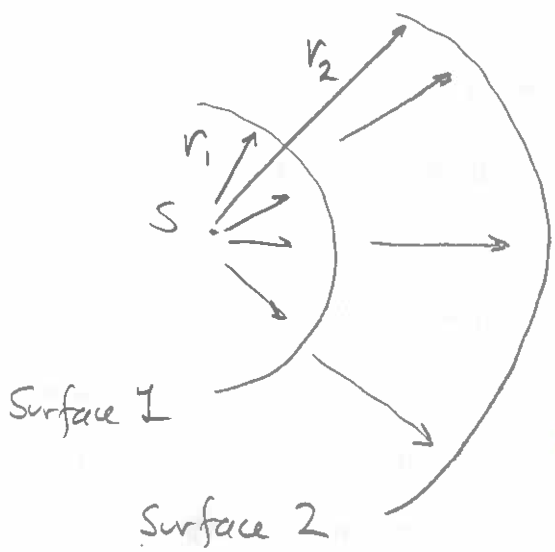
The angle between the incident wave front and the surface is $\theta_i = \sin^{-1}\frac{ct}{ab}$.

The reflected wave source at "a" is now at "d" a distance ct, but the reflected source at b is just starting to radiate so the line db defines an outgoing phase front. Now from similar triangles

$$\theta_0 = \sin^{-1}\frac{ad}{ab} = \sin^{-1}\frac{ct}{ab} = \theta_i \;!$$

Prove that intensity falls as $1/R^2$ in spherical spreading and $1/R$ in cylindrical spreading.

Consider a source that radiates power symmetrically in all directions



If there is no energy loss, then all of the energy passing through surface r_1 also passes through r_2

$$\mathbb{P}(r_1) = \mathbb{P}(r_2)$$

but,

$$\mathbb{P}(r_1) = I_1 \cdot 4\pi r_1^2 = I_2 \cdot 4\pi r_2^2$$

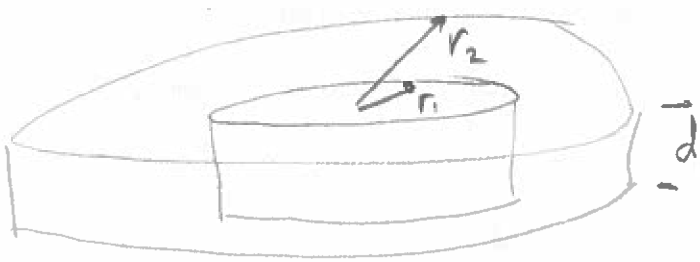
$$I_1 r_1^2 = I_2 r_2^2$$

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$$

If I_1 and r_1 are considered reference range then

$$\boxed{I_2 = \frac{I_1}{r_1^2} \cdot \frac{1}{r_2^2} \propto 1/r_2^2}$$

A similar argument can be applied to a cylindrical geometry:



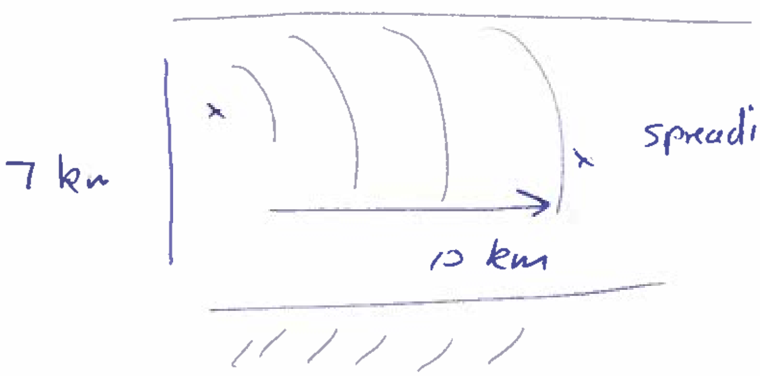
$$\mathbb{P}_1 = \mathbb{P}_2$$

$$I_1 \cdot \cancel{2}\cancel{\pi} r_1 \cancel{d} = I_2 \cdot \cancel{2}\cancel{\pi} r_2 \cancel{d}$$

$$\boxed{I_2 = \frac{I_1}{r_1} \cdot \frac{1}{r_1} \propto 1/r_1}$$

1000 W source, 1000 Hz

7000 m deep ocean



spreading is spherical!

$1.18e^{20} = \frac{I}{I_{ref}}$

Intensity is $\text{Power}/\text{area} = \dfrac{1000\ \text{W}}{4\pi r^2}$ — 200.8 dB or 1.1×10^{4} Pa

r = radius of sphere

$$p^2 = I\cdot\rho c = \frac{1000}{4\pi r^2}\cdot\rho c$$

$$p = \left(\frac{1000}{4\pi r^2}\cdot\rho c\right)^{1/2}$$

but at 1000 Hz, attenuation $\alpha = 1e^{-4}$ dB/m

So,
$$\boxed{p = \left(\frac{1000}{4\pi r^2}\cdot\rho c\right)^{1/2}\cdot 10^{-\alpha r/20}}$$

$$P(10000\,m) = \left[\frac{1000}{4\pi \cdot (10000)^2} \cdot 1025 \cdot 1500\right]^{1/2} \cdot 10^{-1\times10^{-4} \cdot 10^4/20}$$

$$= 0.9776 \text{ Pa}$$

in dB $\rightarrow$ $20 \log_{10} \dfrac{0.9776}{1\times10^{-6}} = \underline{\underline{119.8 \text{ dB}}}$

Repeat the calculation for $50{,}000$ Hz $\Rightarrow$ $\alpha = 1\times10^{-2} \frac{dB}{m}$

$$P(10000) = 1.097\times10^{-5} \text{ Pa}$$

$\Rightarrow$ $20 \log_{10} \dfrac{1.097\times10^{-5}}{1\times10^{-6}} = \underline{\underline{20.8 \text{ dB}}}$

The cylindrical spreading case can be done two ways. The easy way is to take a continuous source so you don't really have to worry about "how" the sound gets to some distance; that's the easy case so we will start there:

$I = \dfrac{\text{Power}}{\text{Area}}$

The problem is the same as for the spherical case But now, area goes as $50\pi r^2$

$$\Rightarrow \quad P = \left(\frac{1000}{50 \cdot 2\pi r} \cdot \rho c\right)^{1/2} \cdot 10^{-\alpha r/20}$$

for 1000 Hz $(\alpha = 1\times10^{-4})$ at 10 km;

$$P = \left(\frac{1000 \quad 1025 \quad 1500}{50 \quad 2\pi r}\right)^{1/2} \cdot 10^{-1\times10^{-4} \cdot 10^4/20}$$

$$= 19.55 \text{ Pa}$$

$$\Rightarrow 20 \log_{10} \frac{19.55}{10^{-6}} = \underline{\underline{145.8 \text{ dB}}}$$

Now for 50,000 Hz $\alpha = 1\times10^{-2}$

$$P = 2.19\times10^{-4}$$

$$\Rightarrow 20 \log_{10} \frac{2.19\times10^{-4}}{10^{-6}} = \underline{\underline{46.8 \text{ dB}}}$$

For the case where the sound is not continuous, you need to consider how the sound makes the transition from spherical to cylindrical spreading:

Out to 50 m, the sound will spread spherically. Beyond that point, cylindrical is more appropriate!

1) Find pressure at 50 m. using spherical, then 2) find pressure at greater distances using cylindrical spreading.

At <u>50 m</u>

$$P_{50} = \left(\frac{1000}{4\pi\, 50^2} \rho c \right)^{1/2} 10^{-\alpha \cdot 50/20}$$

I can use the sonar equation to "propagate" this sound to arbitrary distance:

$$SL_{50} = 20 \log_{10} \frac{P_{50}}{1 \times 10^{-6}}$$

$$SL_r = SL_{50} - \alpha(r-50) - 10 \log_{10} \frac{r}{50}$$

already accounted for loss to 50 m, this will be $r-50 \simeq r$!

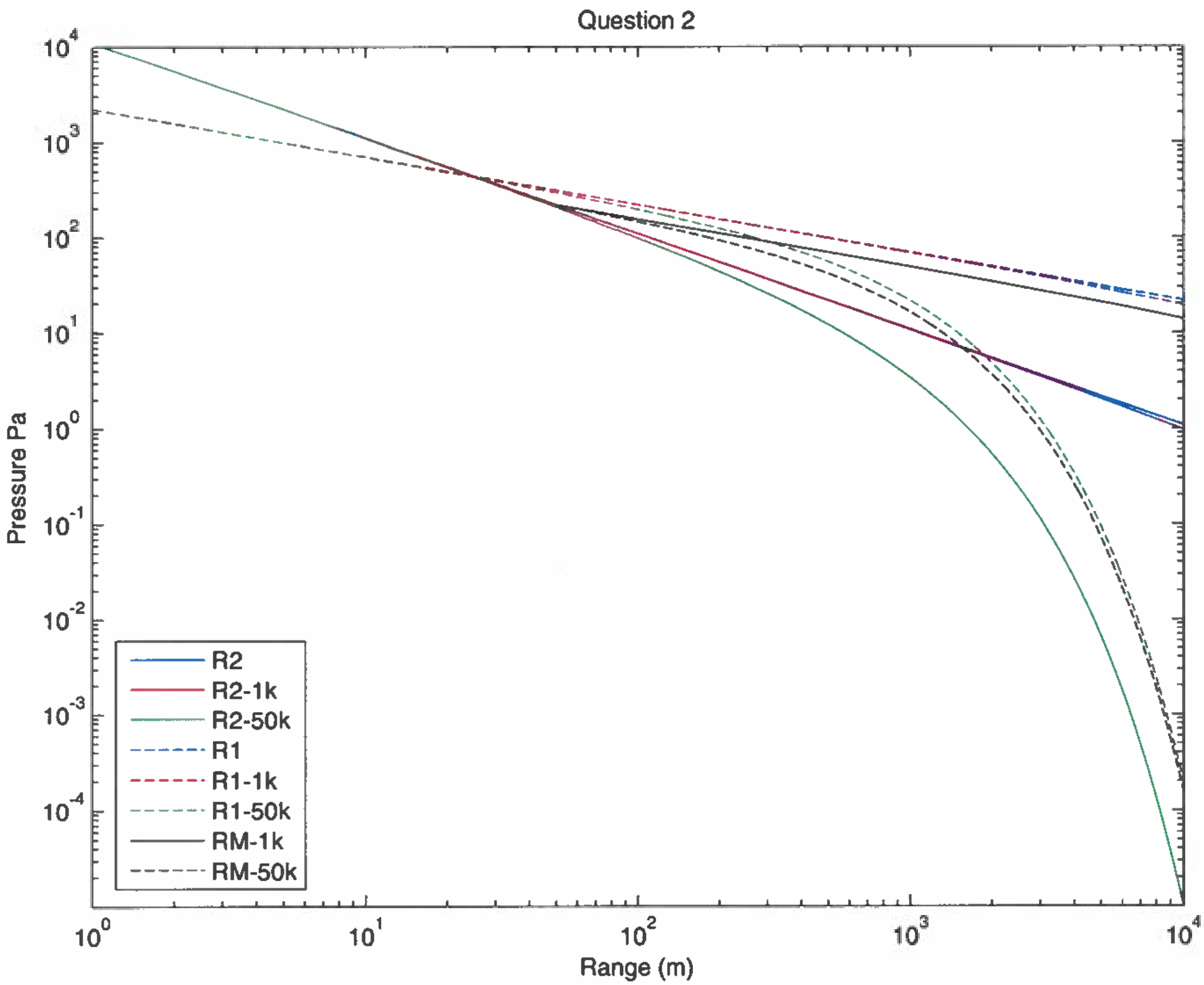
Tricky! this can't be 1 m because the intensity would drop by ½ going to 2 m (52 in our case) R_0 is 50

For 1000 Hz case

$P_{10k} = 13.8$ Pa $\rightarrow$ 142.8 dB

50,000 $\Rightarrow 1.64 \times 10^{-4}$ Pa $\rightarrow$ 44.3 dB

3-6



Question 2
Pressure Pa
Range (m)
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^0
10^1
10^2
10^3
10^4
R2
R2-1k
R2-50k
R1
R1-1k
R1-50k
RM-1k
RM-50k

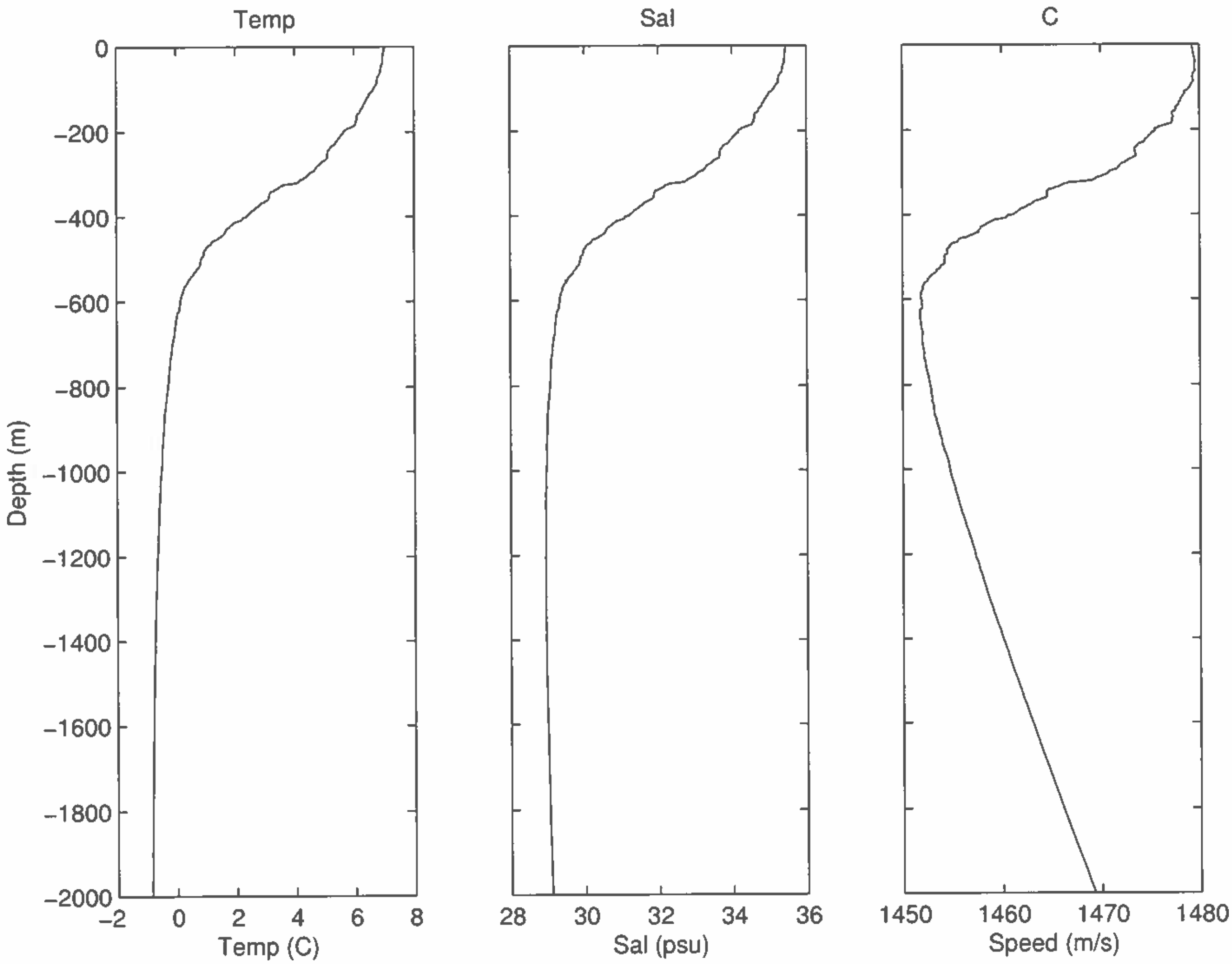
There are really only two important terms in the sound speed relation;

The temperature term: $4.6T$
and the depth term: $0.016Z$

The salinity term is generaly not important because salinity ranges from 30-35 psu and the coefficient of this term is 1.34 $\Rightarrow$ salinity will only change C by 5-10 m/s!

The sound speed profile is dominated by temperature effects in the top 500 m. The temperature decreases 7°C here altering C by ~30 m/s!

Below 500 m, temperature (and salinity) do not change so the depth term $0.016Z$ systematically increases C with depth.


Temp
Sal
C
Depth (m)
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-1800
-2000
-2
0
2
4
6
8
Temp (C)
28
30
32
34
36
Sal (psu)
1450
1460
1470
1480
Speed (m/s)

5-1

There is a set of standard routines for calculating many seawater parameters at: http://www.teos-10.org/. If you search around you should be able to find a routine for sound speed. Plot that version of the sound speed over your result from problem 4. Comment on the differences.

I used routines:

SA = gsw_SA_from_SP(sal,depth,long, lat);   To find the absolute salinity … needed later

For lat and long I guessed at weather station papa: 50 N, 144 W

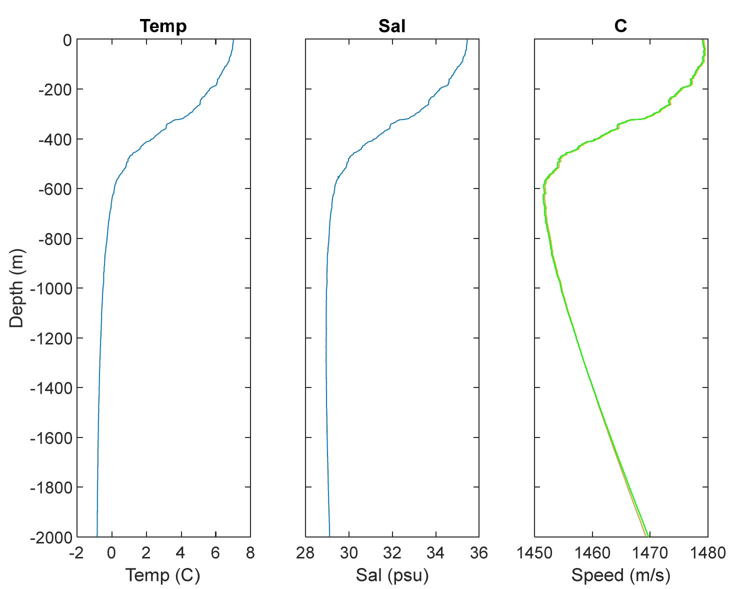
CT = gsw_CT_from_t(SA,temp,depth); To find the conservative temperature

C_teos = gsw_sound_speed(sal,CT,depth);   % sound speed based on 75 term polynomial fit.

AND, two exact formulations that give essentially the same answer:

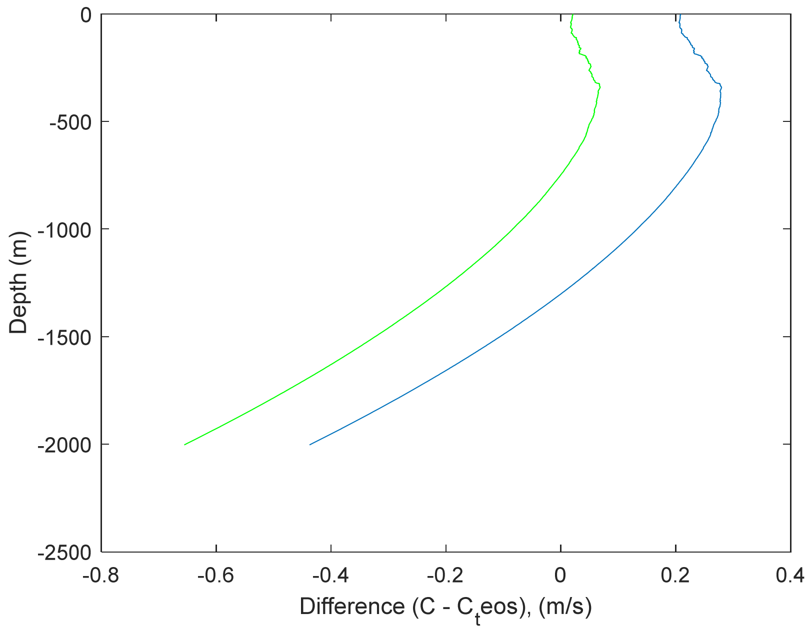
C_exact_t = gsw_sound_speed_t_exact(SA,temp,depth);   %   SA, t, p)
C_exact_CT = gsw_sound_speed_CT_exact(SA,CT,depth);   % SA, CT, p)

First, I plot the C_teos values over top of the original profile based on the Medwin equation (C_teos in green):



5-2

The difference is so small that you need to look at the difference between the two:



Blue is C-Cteos,

The difference in sound speed calculation is less than 0.4 m/s so in fact the Medwin equation is quite accurate and it takes a lot of extra terms to improve the fit.

I repeated the calculation using C_exact which is actually based on C^2 = d \rho / d P and there is a constant offset of about -.2 m/s. I'm not sure where this offset is coming into play …. Likely a result of the lat/long choice which might not be so good.

The polynomial fit is noted to be essentially as good as the thermodynamic solution for almost all applications. You have to keep in mind what accuracy you may need and then look to see what will provide that required accuracy.

Consider $u = u(x - ct)$

as a solution to the wave equation

show that $\frac{\partial u}{\partial t} = -c \frac{\partial u}{\partial x}$

We can write $u = u(s)$ $\quad s = x - ct$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial s}\frac{\partial s}{\partial t} = \frac{\partial u}{\partial s}(-c)$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} = \frac{\partial u}{\partial s}(1)$$

$$-\frac{1}{c}\frac{\partial u}{\partial t} = \frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}$$

giving

$$\boxed{\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x}}$$

as requested!