# P6317 Assignment I

**Due, Monday, October 14, 2019**

1) [5] Use the Huygen's wavelet concept to prove that the angle of incidence equals the angle of reflection for a plane wave reflecting from a planar surface.

2) [5] Prove that sound free to radiate in a three dimensional domain will decay in intensity in proportion to $1/R^2$ while sound constrained to two dimensions will decay in proportion to $1/R$.

3) [10] A point radiates 1000 W of acoustic power at 1000 Hz in a 7000 m deep ocean. Plot a graph of the rms pressure as a function of range from 1 to 10000 m. (Use log-log axes). Repeat the calculation if the sound source is in the arctic where the water is only 50 m deep (ignore any energy loss with bottom and surface interactions). Repeat both calculations assuming a 50000 Hz source. In each case, what would the sound level be in dB (re 1 $\mu$Pa) at a range of 10,000 m.

   (warning, think carefully about whether spherical or cylindrical spreading or some combination of the two is appropriate, passing off between the two requires some caution.)

4) [5] For the CTD profile provided on the ftp site, brigus.physics.mun.ca in directory, /pub/zedel/p6317 in file, ctdprofile.dat, plot profiles of temperature, salinity, and sound speed as a function of depth. (the columns in ctdprofile.dat are; depth (m), temperature (celsius), salinity (psu), and I don't know what the other columns are.) To estimate sound speed use the relation:

$$C = 1449.2 + 4.6T - 0.055T^2 - 0.000291T^3 + (1.34 - 0.01T)(S - 35) + 0.016Z. \quad (1)$$

   Explain the shape of the sound speed profile in relation to the temperature and salinity profiles; what factors are important at what depth intervals.

5) [5] Explain what operating frequency sonar you would choose for the following applications. Explain how your choice is constrained by the application and by the physical limitations of sound in the ocean. (Realise that this question has no "exact" answers but you must justify your choices.)

   i) Echosounder for fish finding in 500 m depth.

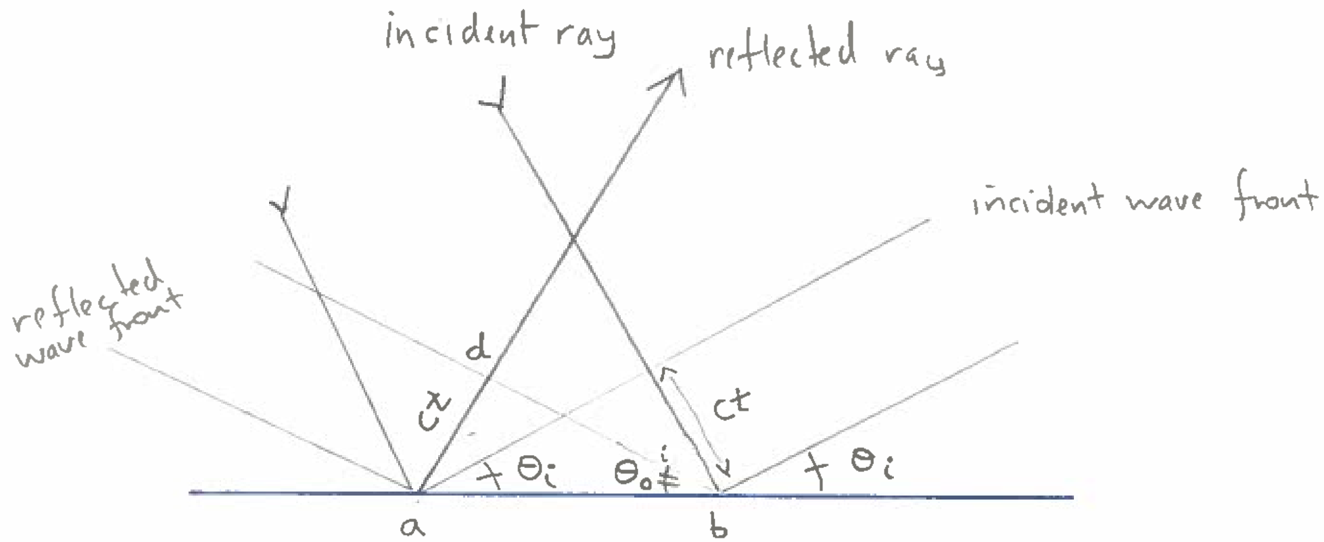
   ii) Echosounder for plankton survey in 100 m depth.

   iii) Sidescan sonar for operation in 100 m water depth looking for sunken galleon (and, of course, treasure hidden within).

   iv) System for communicating over 500 km range.

   v) System for communicating over 500 m range.

1)




At some time, a phase front of the incident plane wave encounters the surface at "a" this point becomes a Huygen's source point. Wavelets radiate from "a" and at some time $t$ later they have traveled a distance $ct$. Also in that time, the incident wave has travelled a distance $ct$ so that it now encounters the interface at "b".

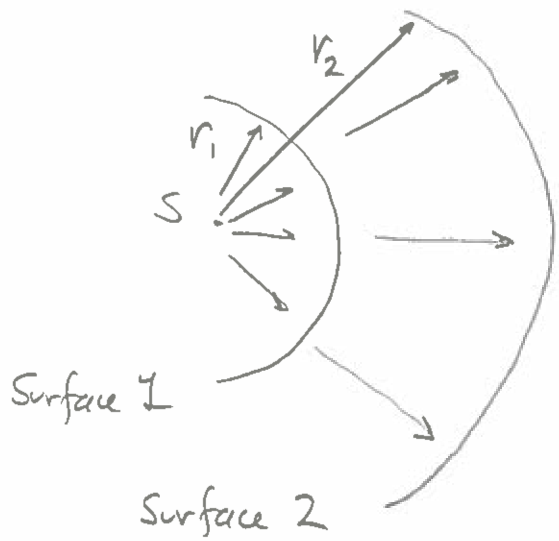
The angle between the incident wave front and the surface is $\theta_i = \sin^{-1}\frac{ct}{ab}$.

The reflected wave source at "a" is now at "d" a distance $ct$, but the reflected source at b is just starting to radiate so the line db defines an outgoing phase front. Now from similar triangles

$$\theta_o = \sin^{-1}\frac{ad}{ab} = \sin^{-1}\frac{ct}{ab} = \theta_i \ !$$

Prove that intensity falls as $1/R^2$ in spherical spreading and $1/R$ in cylindrical spreading.

Consider a source that radiates power symmetrically in all directions




If there is no energy loss, then all of the energy passing through surface $r_1$ also passes through $r_2$

$$\mathbb{P}(r_1) = \mathbb{P}(r_2)$$

but,

$$\mathbb{P}(r_1) = I_1 \cdot 4\pi r_1^2 = I_2 \cdot 4\pi r_2^2$$

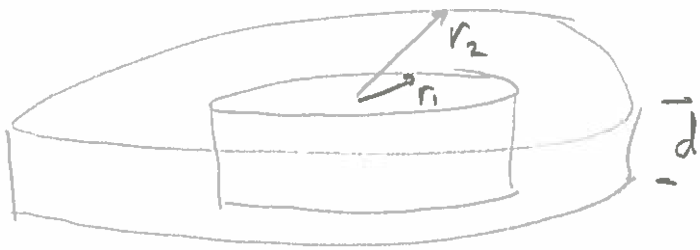
$$I_1 r_1^2 = I_2 r_2^2$$

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$$

If $I_1$ and $r_1$ are considered reference range then

$$\boxed{I_2 = \frac{I_1}{r_1^2} \cdot \frac{1}{r_2^2} \propto 1/r_2^2}$$

A similar argument can be applied to a cylindrical geometry:



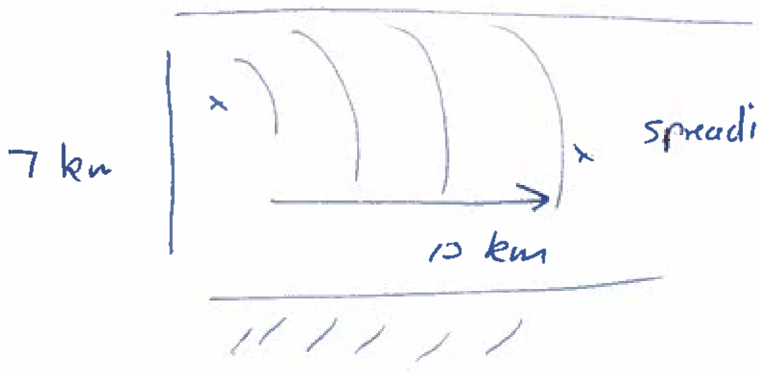

$$\mathbb{P}_1 = \mathbb{P}_2$$

$$I_1 \cdot \cancel{2\pi} r_1 \cancel{d} = I_2 \cdot \cancel{2\pi} r_2 \cancel{d}$$

$$\boxed{I_2 = \frac{I_1}{r_1} \cdot \frac{1}{r_1} \propto 1/r_1}$$

1000 W source, 1000 Hz

7000 m deep ocean




spreading is spherical!

$1.18e^{20} = \frac{I}{I_{ref}}$

Intensity is Power/area $= \frac{1000\ W}{4\pi r^2}$ — 200.8 dB or $1.1\times10^{4}$ Pa

r = radius of sphere

$$p^2 = I \cdot \rho c = \frac{1000}{4\pi r^2} \cdot \rho c$$

$$P = \left(\frac{1000}{4\pi r^2} \cdot \rho c\right)^{1/2}$$

but at 1000 Hz, attenuation $\alpha = 1e^{-4}$ dB/m

So,
$$\boxed{P = \left(\frac{1000}{4\pi r^2} \cdot \rho c\right)^{1/2} \cdot 10^{-\alpha r/20}}$$

$$P(10000\,m) = \left[\frac{1000}{4\pi \cdot (10000)^2} \cdot 1025 \cdot 1500\right]^{1/2} \cdot 10^{-1\times10^{-4} \cdot 10^4/20}$$

$$= 0.9776 \text{ Pa}$$

in dB $\rightarrow$ $20 \log_{10} \frac{0.9776}{1\times10^{-6}} = \underline{\underline{119.8 \text{ dB}}}$

Repeat the calculation for $50{,}000$ Hz $\Rightarrow \alpha = 1\times10^{-2} \frac{dB}{m}$

$$P(10000) = 1.097 \times 10^{-5} \text{ Pa}$$

$$\Rightarrow 20 \log_{10} \frac{1.097\times10^{-5}}{1\times10^{-6}} = \underline{\underline{20.8 \text{ dB}}}$$

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The cylindrical spreading case can be done two ways. The easy way is to take a continuous source so you don't really have to worry about "how" the sound gets to some distance; that's the easy case so we will start here:


$I = \frac{\text{Power}}{\text{Area}}$

The problem is the same as for the spherical case But now, area goes as $50\pi r^2$

$$\Rightarrow \quad P = \left(\frac{1000}{50\cdot 2\pi r}\cdot \rho c\right)^{1/2}\cdot 10^{-\alpha r/20}$$

for 1000 Hz $(\alpha = 1\times10^{-4})$ at 10 km;

$$P = \left(\frac{1000 \quad 1025 \quad 1500}{50 \qquad 2\pi r}\right)^{1/2}\cdot 10^{-1\times10^{-4}\cdot 10^{4}/20}$$

$$= 19.55 \text{ Pa}$$

$$\Rightarrow 20\log_{10}\frac{19.55}{10^{-6}} = \underline{\underline{145.8 \text{ dB}}}$$

Now for 50,000 Hz $\alpha = 1\times10^{-2}$

$$P = 2.19\times10^{-4}$$

$$\Rightarrow 20\log_{10}\frac{2.19\times10^{-4}}{10^{-6}} = \underline{\underline{46.8 \text{ dB}}}$$

For the case where the sound is not continuous, you need to consider how the sound makes the transition from spherical to cylindrical spreading:


Out to 50 m, the sound will spread spherically. Beyond that point, cylindrical is more appropriate!

1) Find pressure at 50 m. using spherical, then 2) find pressure at greater distances using cylindrical spreading.

At <u>50 m</u>

$$P_{50} = \left( \frac{1000}{4\pi\, 50^2} \rho c \right)^{1/2} 10^{-\alpha \cdot 50/20}$$

I can use the sonar equation to "propagate" this sound to arbitrary distance:

$$SL_{50} = 20 \log_{10} \frac{P_{50}}{1 \times 10^{-6}}$$

$$SL_r = SL_{50} - \alpha(r-50) - 10 \log_{10} \frac{r}{50}$$

already accounted for loss to 50 m, this will be $r-50 \simeq r$!

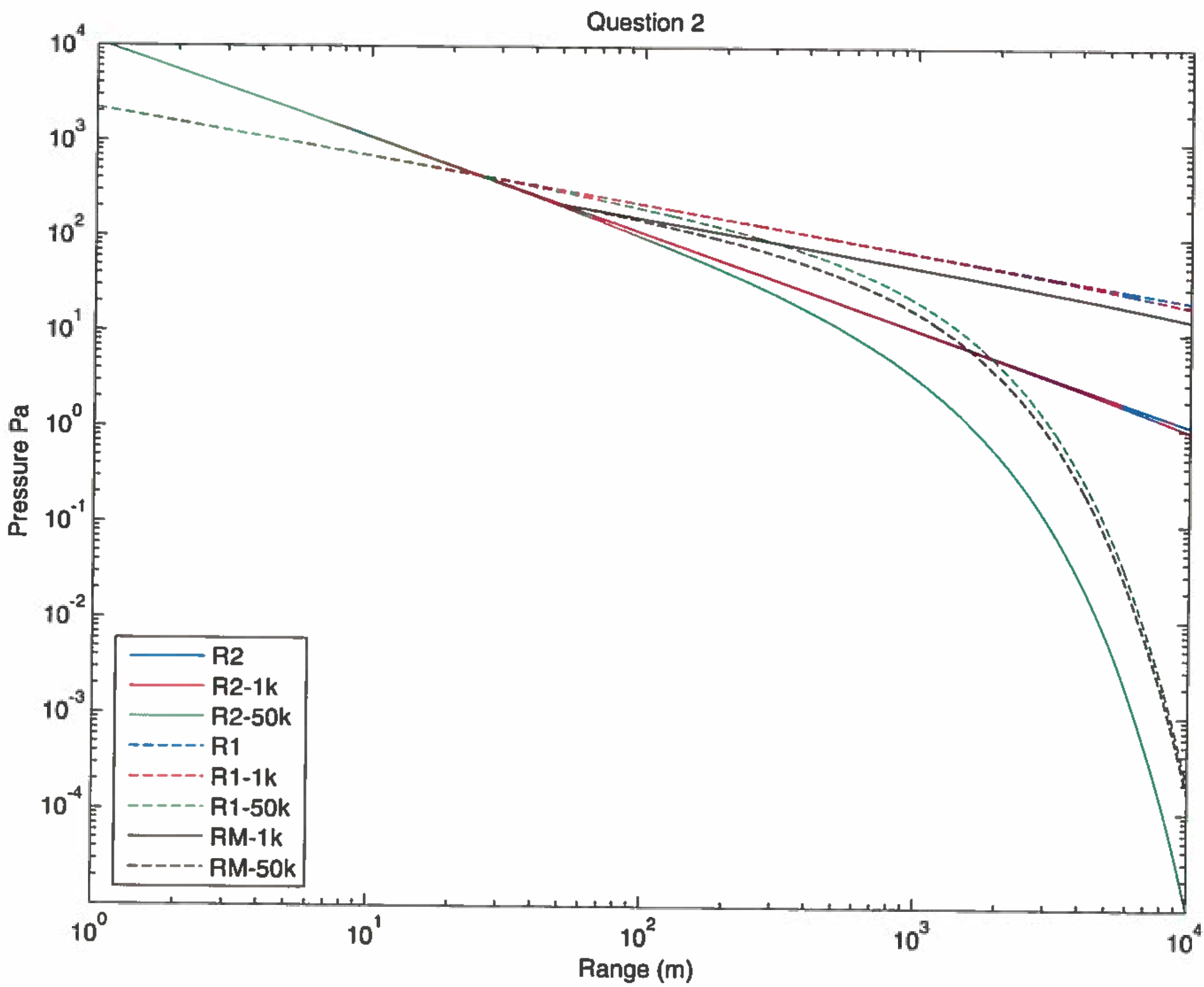
Tricky! this can't be 1m because the intensity would drop by ½ going to 2 m (52 in our case) $R_0$ is 50

For 1000 Hz case

$P_{10k} = 13.8$ Pa $\rightarrow$ 142.8 dB

50,000 $\Rightarrow 1.64 \times 10^{-4}$ Pa $\rightarrow$ 44.3 dB

2-6



Question 2
Pressure Pa
Range (m)
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^0
10^1
10^2
10^3
10^4
R2
R2-1k
R2-50k
R1
R1-1k
R1-50k
RM-1k
RM-50k

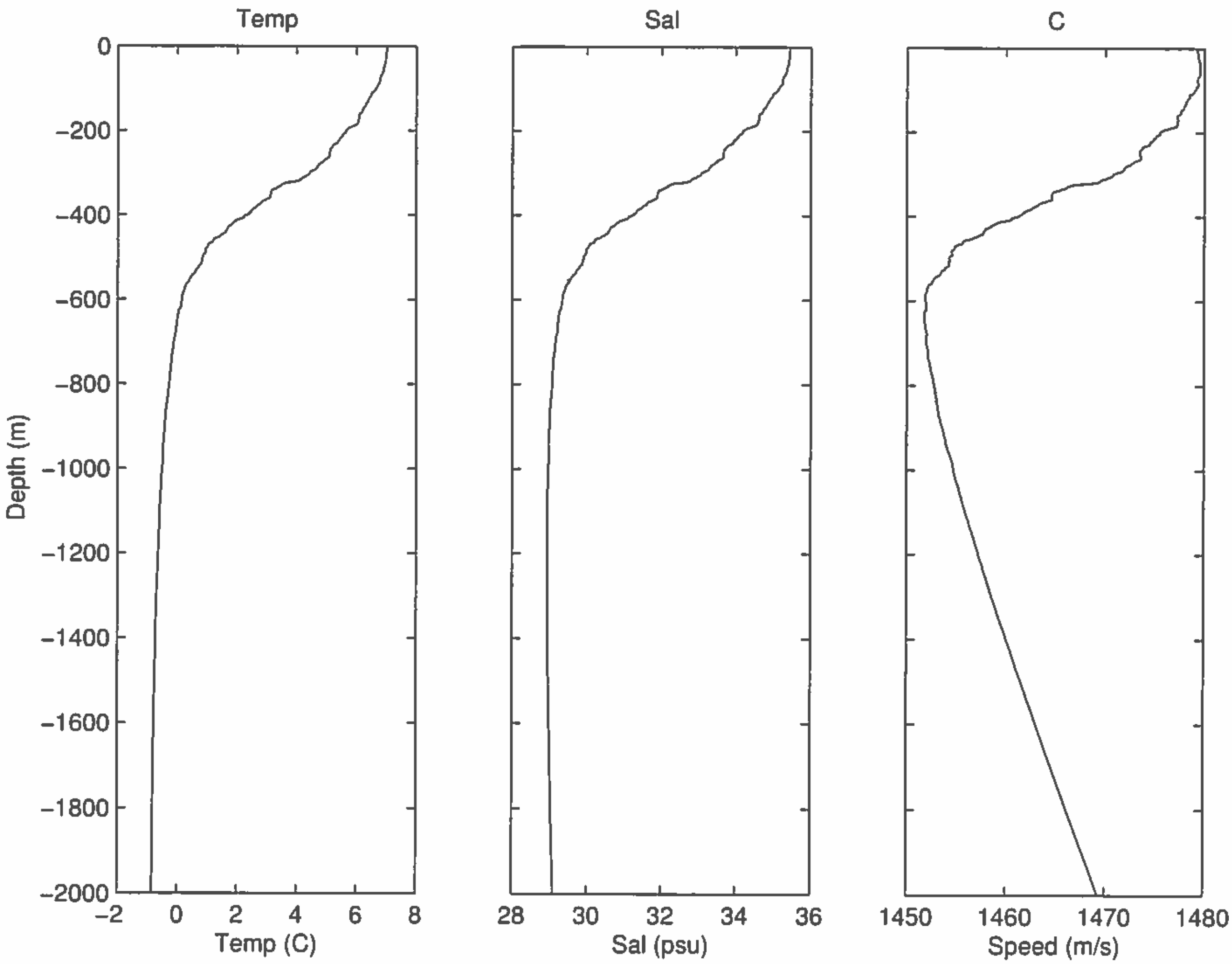
There are really only two important terms in the sound speed relation;

The temperature term: $4.6T$
and the depth term: $0.016Z$

The salinity term is generaly not important because salinity ranges from 30-35 psu and the coefficient of this term is 1.34 $\Rightarrow$ salinity will only change $C$ by 5-10 m/s!

The sound speed profile is dominated by temperature effects in the top 500 m. The temperature decreases 7°C here altering $C$ by ~30 m/s!

Below 500 m, temperature (and salinity) do not change so the depth term $0.016Z$ systematically increases $C$ with depth.


Temp
Sal
C
Depth (m)
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-1800
-2000
-2 0 2 4 6 8
Temp (C)
28 30 32 34 36
Sal (psu)
1450 1460 1470 1480
Speed (m/s)

Operating frequency for sonar.

- the choice is usually a trade off between the highest frequency possible and the maximum range needed. Otherwise you just make sure the frequency is high enough to detect what it is you are looking for.

i) Echosounder for fish at 500m

to get 500 m range need frequency less than about 100 kHz. Fish are big, say 10 cm for the smallest, $ka = 1 = \frac{2\pi}{\lambda} \cdot 0.1 = 1$

Suggests $\lambda \simeq \frac{2\pi}{10} \Rightarrow$ 2500 Hz minimum

so, anything between $2500^{\text{Hz}} \rightarrow$ 100 kHz will work.

ii) Plankton at 100m

using $a = 0.001$ m $\xRightarrow{\text{(as above)}}$ 250 kHz $= f$

$\Rightarrow$ choose $f >$ 250 kHz, the 100m range would require $f <$ about 300 kHz.

iii) Sidescan 100m water. Length scale is probably 10 cm $\Rightarrow$ 2500 Hz is the low end

For nice pictures 100 kHz would be fine

iv) 500 km is a long distance, assuming cylindrical spreading you will lose 57 dB alone so you won't want to lose much more -- say you allow 20 dB ~~to~~ to absorption that requires less than $4 \times 10^{-5}$ dB/m which you get for frequencies around 500 Hz

v) At 500 m range, even spherical spreading will only take $20 \log R = 54$ dB So again lets allow 20 dB to absorption suggesting 0.04 dB/m and you can use 100 kHz