P4820 Assignment IV

Due, April 3 (negotiable!)

1) A solid sphere of radius a is immersed in a vat of fluid at temperature T_o. Heat is conducted into the sphere according to equation

$$\frac{\partial T}{\partial t} = \frac{kA}{mc}\nabla^2 T = D\nabla^2 T$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

If the temperature of the boundary is fixed at T_0, and the initial temperature of the sphere is T_1, find the temperature within the sphere as a function of time. Plot the temperature $(T - T_0)/(T_1 - T_0)$ as a function of radius r for $Dt/a^2 = 1/20,\ 1/10$, and $1/5$.

Hint: The equation in r is "easier" to deal with if you make the change:

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) = r\frac{\partial^2}{\partial r^2}(rR).$$

2) Consider a sphere of radius a for which the top half is set to voltage V and the bottom half is set to voltage 0 (grounded). Find the potential for this sphere for values of $r > a$ (ie. the potential on the outside of the sphere). You will recall that we did this problem solving for the potential on the inside of the sphere in class. Create a plot of the potential (contour?) to demonstrate that your solution makes sense, ie. voltages are appropriate at the surface of the sphere and they go to zero at infinity.

3) A cylinder of height h and radius a has the top and bottom grounded. The potential on the wall at $\rho = a$ is V_0. Find the potential inside the cylinder. Plot $\Phi(\rho, z = h/2)/V_0$ for $0 < \rho < a$ and $\Phi(\rho = a/2, z)/V_0$, for $0 < z < h$.