P4820 Assignment III

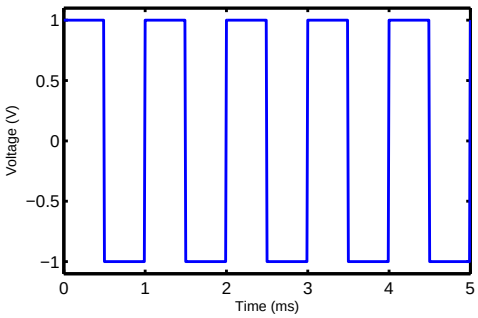
Due, March 8, 2019

1) [5] What distribution is represented by the relation $e^{-x}\,\delta'(x)$?

2a) [5] Find the current as a function of time for an RC circuit driven by a square wave from a function generator as shown below. Kirchoff's loop rule gives:

$$V(t) = q(t)/C + i(t)R \tag{1}$$

where $V(t)$ is the forcing voltage, $q(t)$ is the charge on capacitance C, R is the resistance, and $i(t)$ is the current. Express the CURRENT in the circuit in terms of delta functions, and solve for the current as a function of time. You will need to use the Fourier series representation of the delta function which conveniently repeats periodically. Plot the result for a circuit with R = 100 Ω, C = 2.0×10^{-5} F, and T = 0.001 s;



2b) [10] (Forgive me but the result is a teachable moment) Repeat the circuit analysis using Laplace transforms (just to keep in practice). You can't assume that the initial current is 0 so you need to figure out what it should be by requiring the time average current to be 0!

2c) [5] Graph your results from parts 2a and 2b and compare the two solutions.

3) [10] In class, we considered the differential equation:

$$m\frac{dv}{dt} + \alpha v = F(t) \tag{2}$$

which represents the velocity of an object acted on by some force $F(t)$ where there is a drag term (represented by the αv). and found the Green's function to be:

$$G(t,t') = \begin{cases} 0 & \text{if} \quad t < t' \\ A\mathrm{e}^{-\alpha t/m} & \text{if} \quad t > t' \end{cases}. \tag{3}$$

Find the velocity of the object that results if the forcing term is given as:

$$F(t) = \begin{cases} 0 & \text{if} \quad t < 0 \\ \beta t & \text{if} \quad 0 < t < T \\ 0 & \text{if} \quad t > T \end{cases} \tag{4}$$

Plot the velocity as a function of time given that $\alpha = 1$ Ns/m, $\beta = 1$ N/s, $m = 1$ kg, and $T = 1$. (plot the graph out to t/T = 5).