P4820 Assignment I

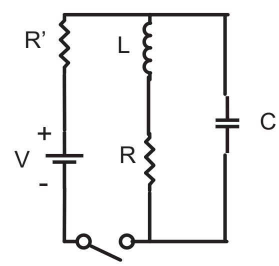
Due, January 28, 2019

1) Use Jordan's Lemma to demonstrate that:

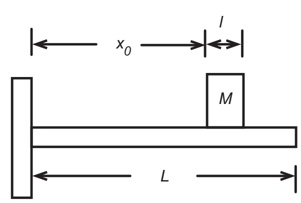
$$\int_{i\infty}^{-i\infty} \frac{3s}{(s+1)(s-3)}\, e^{st} ds = 0.$$

(hint, to make this work you have to use the substitution $s = ip$).

2) The switch in the circuit shown in the diagram below has been closed for a long time, and a constant current flows. What is the charge on the capacitor? At time $t = 0$, the switch is opened. What are the charge on the capacitor and the current through the inductor a long time later? find the current through the inductor as a function of time for $t > 0$. Give your answer in terms of ω_0 and α, where $\omega_0^2 = 1/LC$ and $\alpha = R/2L$.



3) A beam is supported at one end, as shown below. A block of mass M and length l is placed on the beam, as shown. Write down the known conditions at $x = 0$ and $x = L$. Use the Laplace transform to solve for the beam displacement. **WATCH OUT, the way we did it in class, down is positive!.** Plot your results for $y(x)$, $y'(x)$, $y''(x)$, $y'''(x)$ for $x_0 = 0.6L$ and $l = 0.2L$ given $M = 10.$ kg (you can just use arbitrary values for the constants; I used 10, 1000, and 1 for M, E, I respectively). Comment on the nature of the plots and in particular, note the agreement (or failure of agreement) with the expected boundary conditions.



4) The radioactive series that begins with Neptunium 237 contains the following decays:

Decay	Type	Half-life
^{237} Np $\rightarrow$ ^{233} Pa	α	2.14×10^6 y
^{233} Pa $\rightarrow$ ^{233} U	β	27.0 d
^{233} U $\rightarrow$ ^{229} Th	α	1.6×10^5 y
^{229} Th $\rightarrow$ ^{225} Ra	α	7340 y
^{225} Ra $\rightarrow$ ^{225} Ac	β	14.8 d
^{225} Ac $\rightarrow$ ^{221} Fr	α	10.0 d
^{221} Fr $\rightarrow$ ^{217} At	α	4.8 min
^{217} At $\rightarrow$ ^{213} Bi	α	0.032 s
^{213} Bi $\rightarrow$ ^{213} Po (98%)	β	47 min
^{213} Bi $\rightarrow$ ^{209} Tl (2%)	α	
^{213} Po $\rightarrow$ ^{209} Pb	α	4.2 μs
^{209} Tl $\rightarrow$ ^{209} Pb	β	2.2 min
^{209} Pb $\rightarrow$ ^{209} Bi	β	3.3 h

If we regard any decay that takes less than one year to be essentially instantaneous, then the chain simplifies to

$$^{237}\text{Np} \quad \rightarrow \quad ^{233}\text{U} \quad \rightarrow \quad ^{229}\text{Th} \quad \rightarrow \quad ^{209}\text{Bi}.$$

Write a series of differential equations that describes this simplified decay chain. Apply the Laplace transform to find the fraction of the original ^{237}Np that is in the form of uranium, thorium, and bismuth as a function of time. Make a plot (using log-log axes) showing the amounts of each element (^{237}Np, ^{233}U, ^{229}Th, and ^{209}Bi) as a function of time.