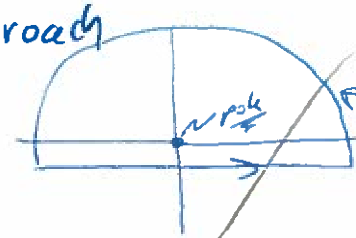
This page JUNK → (Original approach from Lea)



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$$g_{n\pm}(z) = \frac{\mp 2in}{z} + 2n^2 + \ldots$$

Simple pole with residue at $z=0$ $\quad \underline{\mp 2in}$

we go up here so that $e^{2inx} \to 0$ for big

* check this Jordan's Lemma

Consider now the first integral

$$\frac{1}{4n\pi}\int_{-\infty}^{\infty} \frac{1-e^{2inx}}{x^2} f(x)\,dx \quad \left[\frac{c_{-1}}{z} + c_1 + c_2 z + c_3 z\right] \cdot f(z)$$

unresolved!

Danger

gotta show

$$= \frac{1}{4n\pi}\cdot 2\pi i\left[\underbrace{(-2in)\cdot f(0)}_{\text{residue at } z=0} + \sum_p \underbrace{\mathrm{Res}\, f(z_p)\; g_{n+}(z_p)}_{\text{residue of}}\right]$$

$$= f(0) + \frac{i}{2n}\sum_p \mathrm{Res}\, f(z_p)\,\frac{1-e^{2inz_p}}{z_p^2} \to f(0) \quad \text{as } n\to\infty$$

$\to 0$ ~~as $z\to\infty$~~

The second integral has the same path along x but closes in the -ve imaginary domain → thereby excluding the pole!

$$\lim_{n\to\infty}\int_{-\infty}^{\infty}\frac{1}{4n\pi}\frac{\sin^2 nx}{x^2} f(x)\,dx = f(0) \;!$$

* what if $f(z)=0|_{z=0}$? ⇒ could eliminate the pole of g_n!
⇒ $=0$ anyway!
right answer wrong reason!

otherwise argument $\to 0$ at $R\to 0$

* still works for higher order poles but gets messy!

This page Junk

we require that

$$\frac{1-e^{2inx}}{x^2}$$

also requires $\frac{1}{n}\int \frac{1-e^{2inz}}{z} \to 0$ as $n \to \infty$

we are going to evaluate $\frac{i}{2n} \cdot \frac{1-e^{2inz}}{z^2}$ but $\oint$ will only work if this goes $\to 0$ at ∞

$\frac{1}{z^2}$ is no problem,

$$\frac{i e^{2inz}}{2n\, z^2} \to \frac{i e^{2in \cdot R(\cos\theta + i\sin\theta)}}{2n\, R^2 (\cos\theta + i \sin\theta)^2}$$

$$= \frac{i e^{2inR\cos\theta}\, e^{-2nR\sin\theta}}{2n\, R^2 e^{2i\theta}} = \frac{i}{2n} \frac{|e^{2inR\cos\theta}|\, e^{-2nR\sin\theta}}{R^2\, e^{2i\theta}}$$

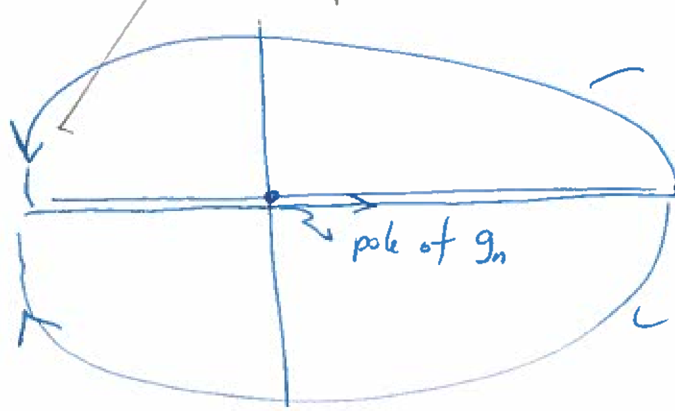
$\to 0$ as $n \to \infty$

$R\sin\theta > 0$ super !

what about

$$\frac{1-e^{-2inx}}{x^2}$$

$$\frac{e^{2inR\cos\theta}\, e^{2nR\sin\theta}}{R^2} \to \text{need } R\sin\theta < 0 \text{ !}$$



for first integral

pole of g_n

second integral (excludes the pole !)